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S-Type Current Controlled New Class Nonlinear Negative Resistances Generation Method

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Abstract. The paper presents a new method for generating a class of S-type nonlinear current controlled negative resistances. The presented method can be used to obtain a large number of nonlinear resistances. We apply the method for a negative resistance device, using bipolar transistors, described by theoretical characteristics and we compare the results with the theoretical specifications. A bipolar negative resistance single mode oscillator model is built. Results are obtained by numerical integration of the differential equations using a new PSPICE method.

1. Introduction

The paper presents a systematic method for generating a class of S-type current-controlled (CC) negative resistance (NR) [1–11]. A considerable research effort exists with regard to the steady-state characteristics of multimode oscillations based on negative resistance. Because of complexity analytical models for nonlinear negative resistance are difficult and rare [1–11].

Section 2 presents the main characteristics of the S-type structures that can be used to design a negative resistance.

In Sections 3 and 4 we present necessary conditions to build a negative resistance (NR) and an example of negative resistance obtained with the presented method. PSPICE computational method for single mode oscillations is presented in Section 5. Finally in Section 6 we present simulation results and conclusions. The presented method can be used to obtain a large number of CC-NR nonlinear resistances.

We apply the method for a negative resistance device, using bipolar transistors, described by theoretical characteristics and performances and we compare the results with the theoretical specifications.

2. S-Type nonlinear CC-NR structures

The presented structures are classified in three categories: Basic Structure (BS); Polarization Structure (PS); Negative Resistance Structure (NRS).

The basic structure of this algorithm is presented in Fig. 1 and can be defined as a three-pole circuit characterized by:

$$v = g\,(i)\,, \quad i > 0. \tag{1}$$

The functions $f\,(v)$ and $g\,(i)$ must have the following properties:

1. Functions $g\,(i)$ must be bijective; and, therefore the inverse function $f\,(v) = g^{-1}\,(i)$ exists;

2. Functions $g\,(i)$ and $f\,(v)$ must be derivable and have the first derivative continuous for $\forall\, i > 0$.

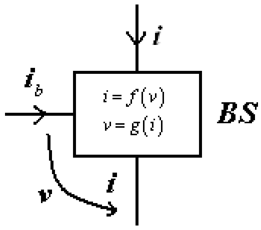

Fig. 1. Basic structure for CC-NR.

This structure can be a BJT, JFET or MOSFET transistor, a circuit with several transistors, an operational amplifier, etc. The general negative resistance structure is shown in Fig. 3. The polarization structures (PS) is presented in Fig. 2 and must satisfy two conditions: the voltage condition and the current condition.

The voltage condition states that:

$$v_2 \cong v_3. \tag{2}$$

This condition is easily fulfilled in practice due to the current sources (Fig. 1).

The current condition can be fulfilled in three ways:

$$i_{b2} = i_{b3}, \tag{3}$$

which is satisfied by using the current sources I_1 as shown in Figs. 2 and 3,

$$i_{b2} \cong 0 \;\text{ and }\; i_{b3} \cong 0, \tag{4}$$

which is satisfied when the base currents are almost negligible (MOS transistors), and

$$i_{b2} \ll i_4 \ \text{ and } \ i_{b3} \ll i_1, \tag{5}$$

which is fulfilled by the structure that uses two bipolar transistors.

Figure 3 presents the negative resistance structure.

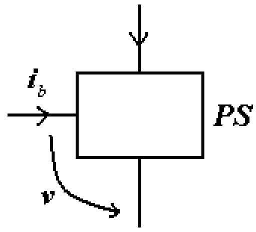

Fig. 2. Polarization structure.

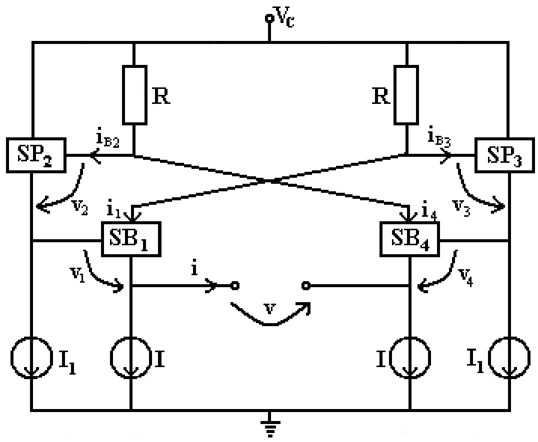

Fig. 3. The negative resistance structures.

From Fig. 3 we can determine:

$$v = v_4 + v_3 + R\,(i_1 + i_{b3} - i_4 - i_{b2}) - v_2 - v_1. \tag{6}$$

Using (2) or (3) or (4) we have:

$$v = v_4 + R\,(i_1 - i_4) - v_1. \tag{7}$$

From Fig. 3:

$$i_1 = I - i > 0 \ ; \ i_4 = I + i > 0. \tag{8}$$

If we attach the characteristically equations for the two base structures:

$$i_{1,4} = f\left(v_{1,4}\right) \text{ or } v_{1,4} = g\left(i_{1,4}\right), \tag{9}$$

we obtain:

$$v\left(i\right) = -2Ri + g\left(I + i\right) - g\left(I - i\right). \tag{10}$$

From (8) we have the condition $|i| < I$. Using now equation (10) we can observe the following properties: Characteristic is zero in origin: $v\left(0\right) = 0$; Characteristic is odd: $v\left(-i\right) = -v\left(i\right)$.

3. Conditions for NR

In (10) if we differentiate $v\left(i\right)$ relative to the current i we obtain the following expression for the negative resistance we search:

$$R_N\left(i\right) = \frac{\mathrm{d}v\left(i\right)}{\mathrm{d}i} = -2R + g'\left(I + i_0\right) + g'\left(I - i_0\right). \tag{11}$$

Usually the negative resistance must have values in a specified interval. We must find the proper value for i that satisfies the following condition:

$$R_N\left(i_0\right) = -2R + g'\left(I + i_0\right) + g'\left(I - i_0\right) < 0. \tag{12}$$

The resistance given by (11) is odd, therefore symmetrical, so if $R_N\left(i\right)$ is negative when $i = i_0$, then it will also be negative for $i = -i_0$.

We can introduce supplementary conditions:

1. It's obvious that $g'\left(I\right) < R$ and $R_N\left(0\right) < 0$.

2. There is only a single peak point in the (I, V) interval, denoted(I_m, V_m). As the characteristic is odd, we have another point, $(-I_m, -V_m)$, that defines a negative resistance.

3. There is also the shape condition: a negative resistance for $i = I$, $i = -I$ is desired, i.e. $v\left(I\right) > 0$ and $v\left(-I\right) < 0$ or equivalently:

$$g\left(2I\right) - g\left(0\right) > 2RI. \tag{13}$$

4. A bipolar negative resistance

The particular structure we present is obtained by using bipolar transistors as is shown in Fig. 4 [5].

The portion between the emitters of T_1 and T_4 may be closed by a capacitor, a series resonant LCR network such as a quartz crystal. We will obtain a nonlinear current-controlled negative resistance (CC_NR) and is typical of commercially available voltage-controlled oscillators (VCOs). The following models describes the transistors T_1 and T_4:

$$i_{1,4} = f\left(v_{1,4}\right) = I_S \exp\left(\frac{v_{1,4}}{V_T}\right),$$

$$v_{1,4} = g(i_{1,4}) = V_T \ln\left(\frac{i_{1,4}}{I_S}\right), \tag{14}$$

$$V_{BE_2} \cong V_{BE_3}, \tag{15}$$

and

$$v(i) = -2Ri + V_T \ln\left(\frac{I_S + i}{I_S - i}\right), \tag{16}$$

where R has units of ohms, I of amperes, and V_T of volts. From (16) we have the condition $|i| < I$, obtained from existence of logarithm function [5].

Using equation (16) we can observe that: the characteristic is zero in origin: $v(0) = 0$ and $v(-i) = -v(i)$.

In (16) if we differentiate $v(i)$ relative to the current i we obtain the following expression for the negative resistance we search:

$$R_N(i) = \frac{\mathrm{d}v(i)}{\mathrm{d}i} = -2R + \frac{2V_T}{I_S}\frac{1}{1 - \left(\frac{i}{I_S}\right)^2},\ |i| < I_S. \tag{17}$$

We obtained a special class of negative resistance.

Observations:

a) CC-NR has a pole in $i = \pm I_S$.

b) For $|i| < I_S$, $R_N(i) = R_N(-i)$.

c) For CC-NR some important values are: $i = 0$, $v(0) = 0$, $R_N(0) = -2\left(R - \frac{V_T}{I_S}\right)$.

d) To obtain a local maximums $R_N(i) = 0$ there

$$I_{m_{1,2}} = \pm I_S \sqrt{1 - \frac{V_T}{RI_S}},$$

$$V_{m_{1,2}} = -2RI_{m_{1,2}} + V_T \ln\left(\frac{I_S + I_{m_{1,2}}}{I_S - I_{m_{1,2}}}\right). \tag{18}$$

To obtain a negative resistance:

$$R_N(0) < 0,$$

or

$$-2R + \frac{2V_T}{I_S} < 0 \text{ and } \frac{V_T}{I_S} < R. \tag{19}$$

This is the condition for current controlled negative resistance existence.

When we will build the oscillator based on the CC-NR using a capacitor, a series resonant LCR network or a quartz crystal we will obtain conditions more restrictive. Conditions can be obtained based on a Kryloff and Bogoliuboff, van der Pol and harmonic balance method [1] using nonlinear ordinary differential equations theory [1–11].

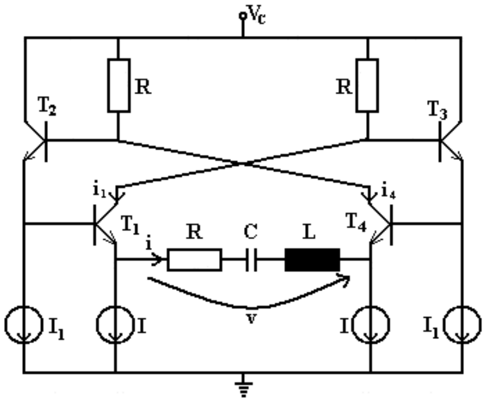

Fig. 4. Negative resistance using bipolar transistors.

5. PSPICE computational method for single mode ECAM

PSPICE has become one of the most well known circuit analysis programs. This paper presents new methods of simulating the algebraic functions as well as the solving of non-linear differential equations.

The following aspects are presented: PSPICE subcircuits achievement for simulating algebraic functions, integrated circuits simulation and non-linear differential equations solving [12].

Results are obtained by numerical integration of the differential equations using a new PSPICE method.

For showing the way that PSPICE works in simulating algebraic functions we will use the circuit from Fig. 5 and equations:

$$E_3 = E_2, V_4 = \ln V_i. \tag{20}$$

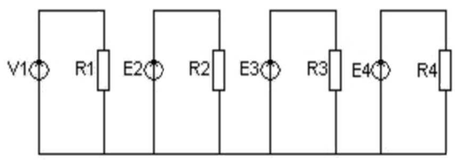

Fig. 5. Function generation subcircuit.

If the voltage commanded source E_4 is:

$$E_4 = 10^{12}\left(E_3 - E_2\right), \text{ then } E_3 - E_2 = \frac{E_4}{10^{12}}. \tag{21}$$

If we consider that E_2, E_3 are much bigger than $E_4/10^{12}$, we obtain:

$$E_3 = E_2. \tag{22}$$

Using PSPICE there can be generated a set of algebraic functions using the circuit from Fig. 5.

This idea is based on the following presumptions:

a) E_2 and E_3 have a polynomial dependence for V_i and E_4;

b) $E_4 = F\left(V_i\right)$.

It is possible that an equilibrium is obtained.

For example, if the output voltage E_4 must be:

$$V_4 = \ln V_i, \text{ or } V_i = e^{V_4}, \tag{23}$$

where V_i is the input voltage, if we impose:

$$E_3 = V_i, \tag{24}$$

we obtain:

$$V_i = e^{V_4}, \text{ or } V_4 = \ln V_i. \tag{25}$$

Using the same method there can be generated also other algebraic functions that can be represented as a polynomial decomposition.

Integration simulation is based on the dependence between the voltage and the current of a capacitor. The circuit that corresponds to this function is represented in Fig. 6. R_4 is used in parallel with the capacitor to allow to achieve the static points at the start of the PSPICE algorithm. Its value must be as big as possible in order to have no influence in the function of the circuit.

The loading current of the capacitor simulated by the current source F_4, commanded by the voltage source V_i (where $R_3 = 10$ kΩ) is:

$$F_4 = \frac{V_i}{R_3}. \tag{26}$$

The V_4 voltage can be written:

$$V_4 = \frac{1}{C_4}\int F_4 \mathrm{d}t. \tag{27}$$

If $E_5 = V_4$ and $C_4 = 100$ µF, eq. (8) becomes:

$$V_5 = \int V_i \mathrm{d}t. \tag{28}$$

With the help of the C_4 capacitor we can control the initial conditions for integration.

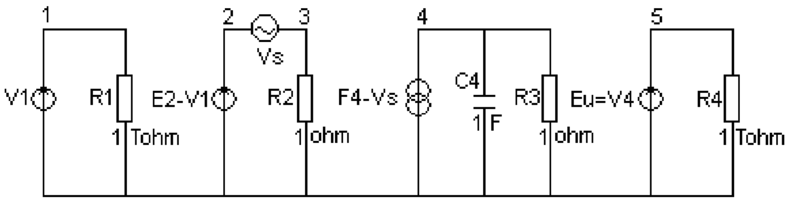

Fig. 6. Integration subcircuit.

In the literature exists a multitude of algorithms for solving the nonlinear differential equations. An approach needs two aspects:

a) a good knowledge of the mathematical algorithm, for choosing the most appropriate algorithm to use;

b) the knowledge of a programming language for the implementation of the algorithm.

The purpose of this paragraph is to present a new method, simple and fast, to solve the nonlinear differential equations using PSPICE.

The equivalent circuit of single-mode is presented in Fig. 7.

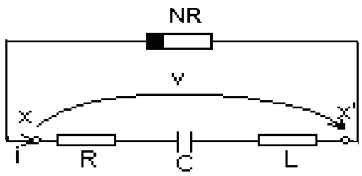

Fig. 7. The single-mode ECAM.

Equation (16) is:

$$v = -a \cdot i + \frac{b}{2} \ln \left(\frac{1 - c \cdot i}{1 + c \cdot i} \right), \tag{29}$$

where:

$$a = 2RC; \quad b = 2V_R; \quad c = \frac{1}{L}. \tag{30}$$

The nonlinear differential equation for the single-mode oscillator circuit is:

$$0 = \frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + \frac{R - a}{L} \left(1 + \frac{bc}{R - a} \cdot \frac{1}{1 - c^2 i^2} \right) \frac{\mathrm{d}i}{\mathrm{d}t} + \frac{i}{LC}. \tag{31}$$

With the notation,

$$\omega^2 = \frac{1}{LC}, \quad \beta = \frac{bc}{a - R}, \quad \varepsilon = \frac{a - R}{\sqrt{\frac{L}{C}}} \tag{32}$$

and the changing of variables:

$$i \to \frac{X}{C}; t \to \frac{t}{\omega}, \tag{33}$$

the relation (31) becomes:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - \varepsilon \left(1 - \frac{\beta}{1 - x^2}\right) \frac{\mathrm{d}x}{\mathrm{d}t} + x = 0 \tag{34}$$

and can be written:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = E_a + E_b + E_c E_a = -x^2 \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}; \tag{35}$$

$$E_b = \varepsilon \left(1 - \beta - x^2\right) \frac{\mathrm{d}x}{\mathrm{d}t}; \quad E_c = x \left(1 - x^2\right). \tag{36}$$

PSPICE has become the standard computer program for most electrical simulation. Higher-level abstraction and hierarchy can be modeled using controlled sources and subcircuits blocks. The nonlinear function applies only to the time domain. PSPICE supports the polynomial sources. Functional models for single mode is presented in Fig. 8.

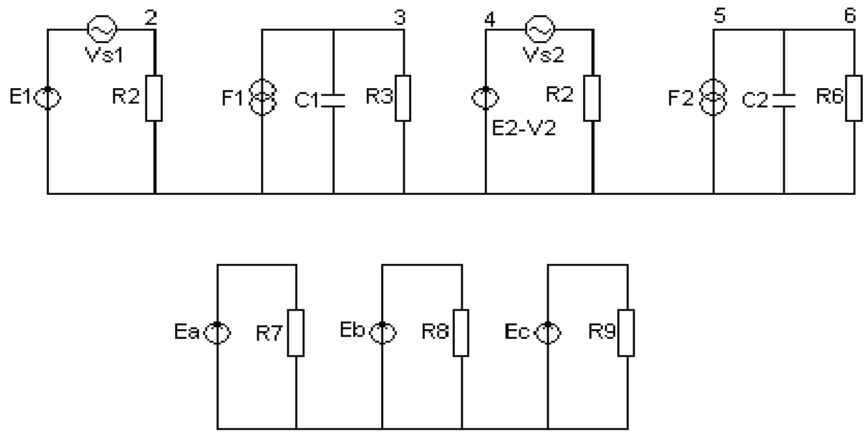

Fig. 8. PSPICE equivalent scheme for single mode nonlinear differential equation.

The integrator (INT blocks in Fig. 6):

$$V_c(t) = \frac{1}{C} \int i_c(t)\, \mathrm{d}t + v_\infty \tag{37}$$

is used in PSPICE to model the capacitor. If

$$i_c(t) = \frac{V_i}{R}, \tag{38}$$

with $R = 10$ kΩ and $C = 100$ µF:

$$V_c(t) = \int V_i \mathrm{d}t + v_\infty. \tag{39}$$

If we have (EQ blocks):

$$E_3 - E_2 = \frac{E_4}{10^{12}} \tag{40}$$

and:

$$E_{2,3} \ll E_4, \quad E_4 = 10^{12}(E_3 - E_2) \tag{41}$$

giving:

$$E_3 = E_2. \tag{42}$$

6. Results and conclusions

The SPICE simulation result proves the theory discussed. The curve is shown for $R = 488$ Ω, I = 1.6 mA, and $V_T = 26$ mV. The characteristic is represented for (–1.6 mA, +1.6 mA) in Fig. 9.

$$v(i) = -976 \cdot i + 0.026 \cdot \ln\left(\frac{0.0016 + i}{0.0016 - i}\right)[V] \tag{43}$$

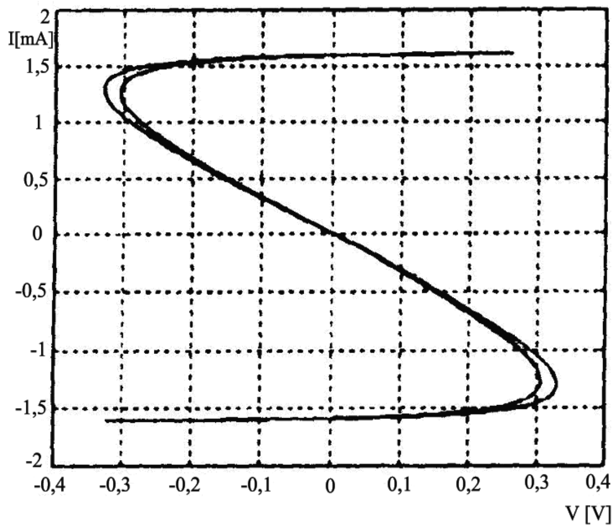

Fig. 9. Bipolar S-type CC-NR characteristic.

From Fig. 9 we can observe that the shape $I-V$ characteristic of bipolar transistor is approaching with a theoretical one. The maximum simulation error for our example using bipolar transistors is 8%.

Based on clear configuration, using polarization and based structures we build a current controlled S-type negative resistance.

In this paper were introduced a new and improved PSPICE method for simulating linear and nonlinear equations. A PSPICE method has been proposed to solve the nonlinear differential equations.

Using the PSPICE program we obtain numerical results. Over 200 runs with several initial conditions prove that we can have a stable oscillation for single mode.

References

[1] VIZIREANU D. N., *A Class of MOS Non-linear Negative Resistance Oscillators*, *IEEE Conference ISCAS '96*, Atlanta, Georgia, USA, 12–15 May 1996, pp. 1134.

[2] VIZIREANU D. N., *A new class of MOS transistor nonlinear negative resistance*, *IEEE Conference on Industrial Applications in Power Systems, Computer Science and Telecommunications Melecon '96*, 13–16 May 1996, pp. 407–411, Italy.

[3] VIZIREANU D. N., HALUNGA S., *NE 564 PLL Skipping Frequency Phenomenon*, *Simpozionul Internaţional Sisteme de comunicaţii mobile*, 1996, Academia Militară, Bucureşti, România.

[4] CHUA L.O., YU J., YU Y., *Bipolar JFET MOSFET Negative Resistance Devices*, *IEEE Transactions on Circuit and Systems*, V-CAS **32**, No. 1, January 1985, pp. 46–61.

[5] WALKER S., CONNELLY J., *A New Negative Resistance Model*, Circuits, Systems and Signal Processing, **2**, 1983.

[6] MICKENS R. E, *Investigation of the mathematical properties of a new negative resistance oscillator model*, Circuits, Systems and Signal Processing, **2**, 1989.

[7] CHENG X., DUANE R., *0.6 V D flip-flop utilising negative differential resistance device*, Electronics Letters, vol. **42**, issue 7, 30 March 2006, pp. 388–390.

[8] RODRIGUEZ S.A, RODRIGUEZ R.J., GONZALEZ J.L.V., *Stability Analysis in a Transmission Line With Nonlinear Elements Using Lyapunov's Method Electronics*, Comm. and Computers, 2006; CONIELECOMP 2006, 16^{th} *Intern. Conf.*, 27 Jan.–01 Feb. 2006, pp. 20–24.

[9] HAYAKAWA Y., HONGGE LI, NAKAJIMA K., *Retrieval property of associative memory with negative resistance*, *IJCNN '05*, *Proc. 2005 IEEE Intern. Joint Conf.*, vol. **2**, 31 July–4 Aug. 2005, pp. 1187–1192.

[10] YOUNG-SANG JEON, HOE-SUNG YANG, SANGWOOK NAM, *A novel EER structure for reducing complexity using negative resistance amplifier*, Microwave and Wireless Components Letters, IEEE, vol. **14**, issue 5, May 2004, pp. 195–197.

[11] MADSEN P., LARSEN T., MIKKELSEN J.H., LINDOF J.C., *An RF CMOS differential negative resistance for Q-enhancement*, *Solid-State and Integrated Circuits Technology*, 2004, *Proc.* 7^{th} *Intern. Conf.*, vol. **2**, 18–21 Oct. 2004, pp. 1256–1259.

[12] ***, *PSPICE*, Microsim Corporation, Irvine, SUA, 1992.