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Influence of the excitation modes on the resonators quality factor

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Abstract.

In this paper, the effect of the acting electrode position on the dynamic behavior of a polysilicon microcantilever is investigated by considering different oscillating modes. The oscillation of structures in higher bending modes is used to increase the quality factor and to decrease the loss of energy during oscillation. The tests are performed in ambient conditions as well as in vacuum. The position of the acting electrode changes the velocity and amplitude of oscillations. A significant influence is determined on the quality factor and on the loss of energy, those are improved if the structures vibrate in the second or third bending modes for the same position of the acting electrode. As the acting electrode position is moved from the cantilever free-end toward to the anchor, the resonator oscillates to a new deflected position and the quality factor is changed. The results are useful for MEMS designers because the sensor/actuator response can be modified only by changing the position of the acting electrode without any significant modification on the fabrication process.

Key-words: Resonator, electrode position, frequency response, quality factor.

1. Introduction

Microelectromechanical resonators are usually fabricated as a simple mechanical structure like a microbridge or a microcantilever that oscillates in the response of an electrostatic actuation signal. The electrostatic actuation represents the most common type of electromechanical energy conversion scheme in microelectromechanical systems. The investigated structures from this paper are polysilicon cantilevers under an electrostatic actuation with different position of the lower electrode. In a microresonator under electrostatic actuation, the dynamic behavior is influenced by the geometrical configurations, the operating conditions, the applied voltages as well as the actuating electrode position. The lower electrode modifies the dynamic response of a microresonators by changing the position of the applied force. This method, to drive the mechanical resonator with an electrode applied close to anchor, is known as the leveraged bending actuation [1, 2, 3, 4]. The leveraged bending is a simple technique to increase the travel distance before the pull-in instability of the resonator by applying an electrostatic force close to resonator anchor. In this case, the remaining portion of structure acts as a lever and can perform large displacement through the entire gap between electrodes. The electrode position changes not only the frequency response, amplitude and velocity of oscillations but also the loss of energy in the vibrating structure which can be experimentally determined by measuring the quality factor of vibrations [2, 5, 6].

The scope of this paper is to determine the electrode position effect on the dynamic response of a microcantilever and to measure the quality factor (Q- factor) under different excitation modes. The ratio between the energy stored in a resonator to the energy dissipated during one cycle of oscillation represent the Q- factor that must be as higher as possible for a resonator in order to be useful in vibrating applications as filter, vibration sensor, mass detection sensor. The Q- factor can be increased by ensuring the functioning of the resonator in a higher mode of vibration [7, 8]. Comparatively with the first bending mode the loss of energy during oscillations is significantly decreased using the 2^{nd} or the 3^{rd} bending mode.

2. Dynamic response of a resonator as a function of the electrode position

2.1. Theoretical aspects

MEMS resonators considered here are polysilicon microcantilevers. The samples are loading by an electrostatic force set up when a voltage is applied between the vibrating cantilever and the lower electrode (Fig. 1).

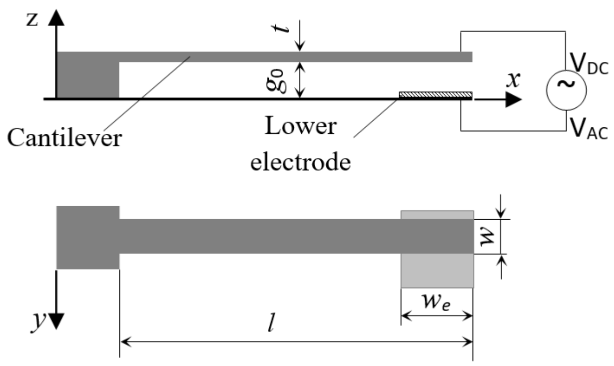


Fig. 1. Schematic representation of an electrostatically actuated MEMS cantilever.

When a DC voltage (V_{DC}) is applied between electrode and cantilever, the cantilever bends downwards and come to rest in a new position. To drive the resonator at resonance, an AC harmonic load of amplitude V_{AC} vibrates the cantilever at the new deflected position. A single degree of freedom model is used to analyze the dynamic response of microresonator due to the V_{DC} and V_{AC} electric loadings as shown in Fig. 2.

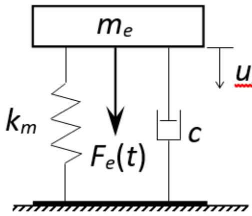


Fig. 2. A single degree of freedom model for electrostatically actuated MEMS cantilever.

In this model the proof mass of microcantilever is modeled as a lumped mass m_e, and its stiffness is considered as a spring constant k_m. This part forms one side of a variable capacitor – the movable part. The bottom electrode is fixed and considered as the second part of the sensor. If a voltage composed of DC and AC terms as

$$V(t) = V_{DC} + V_{AC}\cos(\omega t) \tag{1}$$

is applied between electrodes, the electrostatic force applied on the structure has a DC component as well as a harmonic component with the driving frequency such as:

$$F_e(t) = \frac{\varepsilon A V(t)^2}{2[g_0 - u_z(t)]^2} \tag{2}$$

where ε is the permittivity of the free space, A=w$_e$×w is the effective area of the capacitor, g_0 is the initial gap between flexible plate and substrate, and u_z(t) is the displacement of the mobile plate.

When only a DC voltage is applied across the plates (V_{AC} = 0), the static force balance equation, including the electrostatic force and the spring force is:

$$k_m u_z = \frac{\varepsilon A V_{DC}^2}{2(g_0 - g_z)^2} \tag{3}$$

where u_z is the static displacement of the beam under a DC signal and km is the mechanical stiffness given by:

$$k_m = \frac{EI_y}{l^3} \tag{4}$$

where E is the Young's modulus of cantilever material, I_y is the axial moment of inertia and l is the length of beam.

The equivalent stiffness k_e and resonant frequency ω_0 of microresonator under a DC actuation is obtained by linearizing the electric system around an equilibrium position $\tilde{u}_z$ as:

$$k_e = \frac{3EI_y}{l^3} - \frac{\varepsilon A V_{DC}^2}{(g - \tilde{u}_z)^3} \tag{5}$$

$$\omega_0 = \frac{1}{2\pi}\sqrt{\frac{k_m - \frac{\varepsilon A V_{DC}^2}{(g-\tilde{u}_z)^3}}{m_e}} \tag{6}$$

where m_e, the equivalent mass of system can be calculated using $m_e = 33m/140$, with m - the effective mass of beam.

The free response of a mechanical resonator determines the resonant frequency in either the presence or the absence of damping. The forced response reveals the behavior of an undamped or damped mechanical system under the action of a harmonic excitation. In mechanical resonators, the phenomenon of resonance is important, and in such situation the excitation frequency matches the resonant frequency of the system.

The dynamic response of the microcantilever resonator shown in Fig. 1 subjected to a harmonic electrostatic force $F_e(t)$ with a driving frequency ω given by an AC voltage is governed by the equation of motion:

$$m \cdot \ddot{u}_z(t) + c \cdot \dot{u}_z(t) + k_m \cdot u_z(t) = F_e(t) \tag{7}$$

where $u_z(t)$ is the amplitude of beam oscillations and c is the damping factor. The response of system under DC and AC voltages is given by equation [9]:

$$u_z(t) = \frac{u_z}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_0}\right)^2}} \tag{8}$$

where ξ is the damping ratio and ω_0 is the resonant frequency of microcantilever given by equation (6).

The damping ratio ξ is any positive real number. For value of the damping ratio $0 \leq \xi < 1$, the system has an oscillatory response. The system damping controls the amplitude and velocity of the response when is excited at resonance.

Usually, the response is plotted as a normalized quantity $u_z(t)/u_z$. When the driving frequency equals the resonant frequency $\omega = \omega_0$, the amplitude ratio reaches a maximum value. At resonance, the amplitude ratio becomes:

$$\frac{u_z(t)}{u_z} = \frac{1}{2\xi} \tag{9}$$

An important qualifier of mechanical microresonators is the quality factor Q. At resonance, the quality factor is expressed as [9]:

$$Q_r = \frac{1}{2\xi} \tag{10}$$

and the normalized response given by equation (9) is exactly equal with Q_r.

The quality factor is also called sharpness at resonance, which is defined as the ratio

$$Q_r = \frac{\omega}{\Delta\omega} = \frac{\omega}{\omega_2 - \omega_1} \tag{11}$$

where $\Delta\omega = \omega_2 - \omega_1$ is the frequency bandwidth corresponding to $u_z(t)_{max}\sqrt{2}$ on the amplitude (velocity) versus resonant frequency curves.

2.2. Experimental investigations

The scope of experimental investigation is to analyze the effect of the electrode position on the dynamic behavior of a resonator for different excitation modes. The resonant frequencies, the velocity and amplitude of oscillations are monitored for samples operating in ambient conditions as well as in vacuum. Moreover, the quality factor for investigated samples is determined in different oscillating modes. The experimental dynamic tests are performed using a Polytec Scanning Laser Doppler Vibrometer.

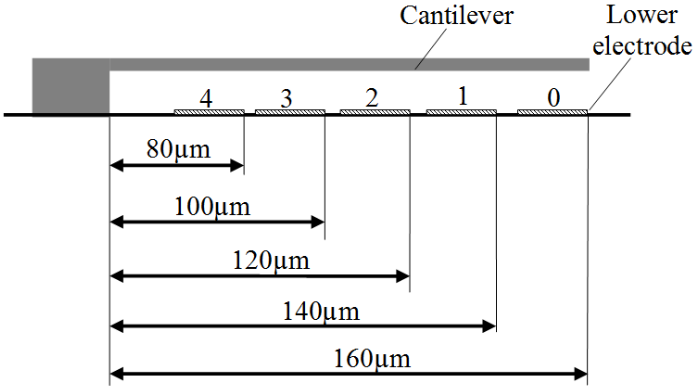


Fig. 3. Successive positions of the lower electrode.

The lower bending moment on cantilever decreases for the same input voltage if the acting electrode is moved from the cantilever free-end toward the beam anchor (Fig. 3). Moreover, the bending pre-stress deflection of beam decreases to the other value depending on the distance between anchor and the applied electrostatic force. This small change in the equilibrium deflection of the beam reduces the efficiency of the electrostatic force and increases the total effective stiffness of system around the equilibrium provided by DC voltage.

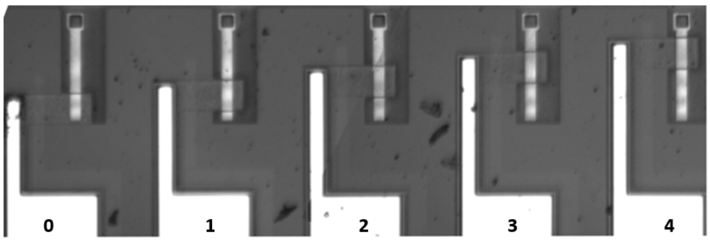


Fig. 4. Resonator microcantilevers with electrostatic actuation and different position of the acting electrode.

Therefore, depending on the position of the electrode, different equilibrium positions, effective stiffness and frequency responses of the beams are obtained for the same impute voltage.

The investigated microcantilevers (Fig. 4) are fabricated from polysilicon with different positions of the lower electrode. The gap between the flexible structure and the acting electrode is 2μm. The width of the lower electrode is $w_e = 50\mu m$. The microcantilever is fabricated with 160μm the length and 1.5μm the thickness. The distance from the cantilever anchor to the lower electrode (Fig.3) is modified from 160μ m (position - 0) to 140μm (position - 1), 120μm (position - 2), 100μm (position - 3) and 80μm (position - 4), respectively.

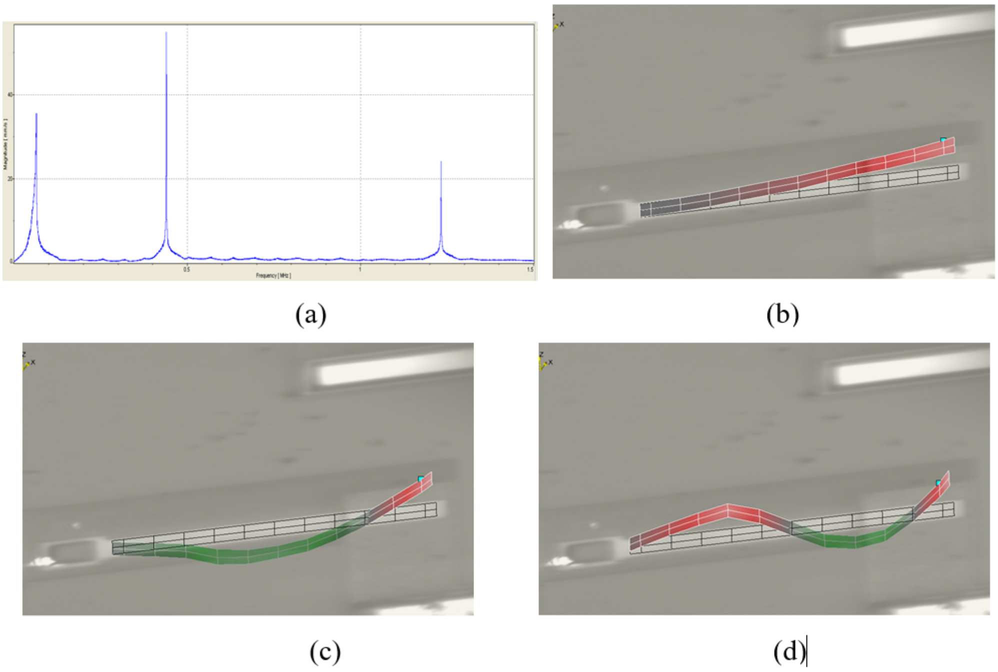


Fig. 5. Dynamic response of the polysilicon microcantilever (position – 0) under different bending modes tested in vacuum (7×10^{-4}mbar): (a) the frequency response curve; (b) the 1st mode; (c) the 2nd mode; (d) the 3rd mode (color online).

During experimental tests a DC offset signal of 5V and the peak amplitude of 5V of the driving signal are applied to bend and oscillate the samples. Different vibration modes of cantilevers can be visualized and analyzed. Figure 5a shows different bending modes for the first cantilever with the lower electrode at the beam free-end (position- 0) tested in vacuum. The first bending mode corresponds to a resonant frequency of 64kHz (Fig.5b), the second bending mode appears at 439.37kHz (Fig. 5c) and the third bending mode is for a resonant frequency equal to 1232.6kHz (Fig. 5d).

Based on the experimental curve of resonant frequency, the Q- factor can be experimentally determined for different bending modes as the ratio between the frequency corresponding to each mode and the measured frequency band width $\Delta\omega$ (Fig. 6). The band width of oscillation $\Delta\omega$ is given by the difference between the maximum resonant frequency and the minimum resonant frequency corresponding to the 0.707 from the maximum amplitude of oscillation.

In order, to be able to determine the Q- factor in vacuum based on the experimental curve, a zoom of the peak of each frequency response mode is needed to allow the band width measurement. Figure 6a presents the zoom of the resonant frequency peak for first bending mode of the cantilever with the acting electrode at the free-end (position- 0). Using the resonant frequency equal of 64kHz and the band width equal to 2 kHz, the Q-factor is determined to 32 in the case of the first bending mode of oscillations. In the same testing conditions, the Q- factor increases to 739.28 for the 2nd bending mode of oscillations (Fig. 6b), and to 1540.75 in the case of the 3rd bending mode (Fig. 6c).

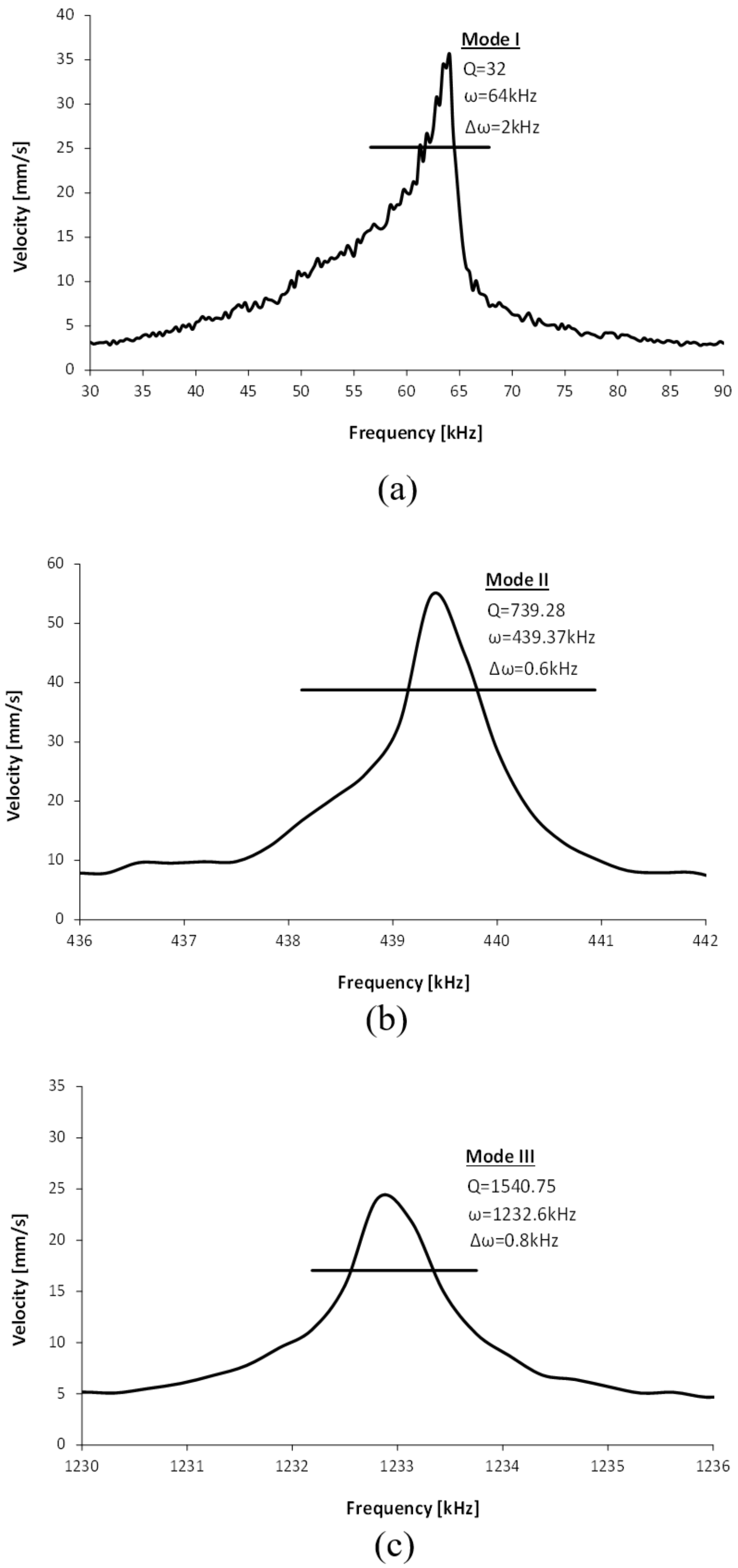


Fig. 6. Zoom of the frequency response and the Q- factor estimation for different bending modes of the cantilever tested in vacuum with the acting electrode at the free-end

For the same sample and under the same electrostatic force the test is repeated in ambient condition. The shape of experimental curve of cantilever with the acting electrode at the beam free-end tested in air (Fig. 7) is different compared to the response in vacuum. This difference is based on the air damping that is a predominant loss of energy parameter of samples oscillated in ambient conditions. Measuring the resonant frequency and the band width the Q- factor can be determined for each bending modes. For the 1st bending mode the Q- factor is determined in a value of 0.71, it increases to 5.30 for the 2nd mode at to 11.78 in the case of the 3rd bending mode of oscillations.

In vacuum, the damping effect depends on the medium pressure. As the vacuum pressure decreases, the Q- factor is improved that increase the velocity and amplitude of oscillations.

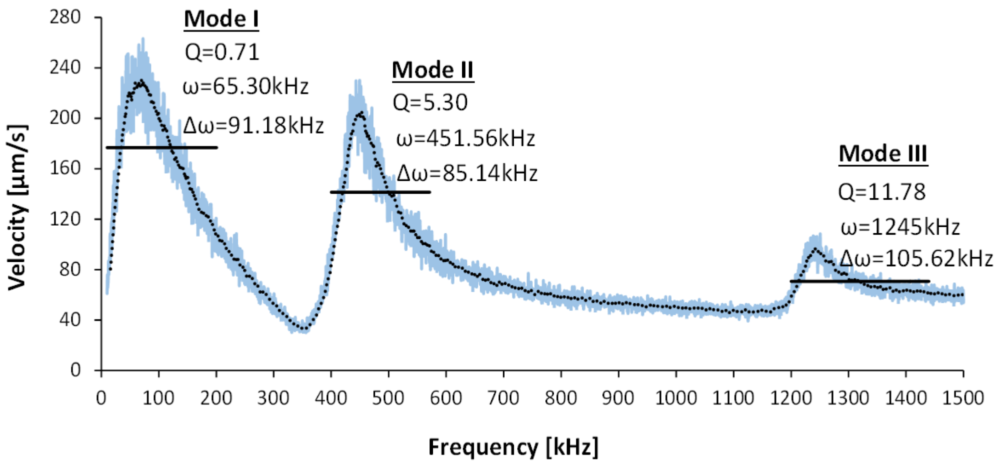


Fig. 7. Frequency response and the Q- factor for different bending modes of the cantilever with position- 0 of the acting electrode tested in ambient conditions.

2.3. Numerical simulations

To validate the experimental values of the dynamic parameters of investigated microcantilever numerical simulations by Finite Element Analysis (FEA) were performed. Pre-stressed modal analysis is used to simulate the frequency response of microcantilever under electromechanical coupling using ANSYS Workbench 17.0. Using the dimension of sample and possitions of acting electrode from experimental investigations (Fig. 3), modeling and finite element analysis of the investigated microcantilever is carried out. A calculated electrostatic force of 24.89 nN, computing using eq. (2) and the geometrical dimensions from experimental tests, is initially applied to calculate the pre-stressed state of the cantilevers under a voltage of 5V (DC), followed by the modal solution.

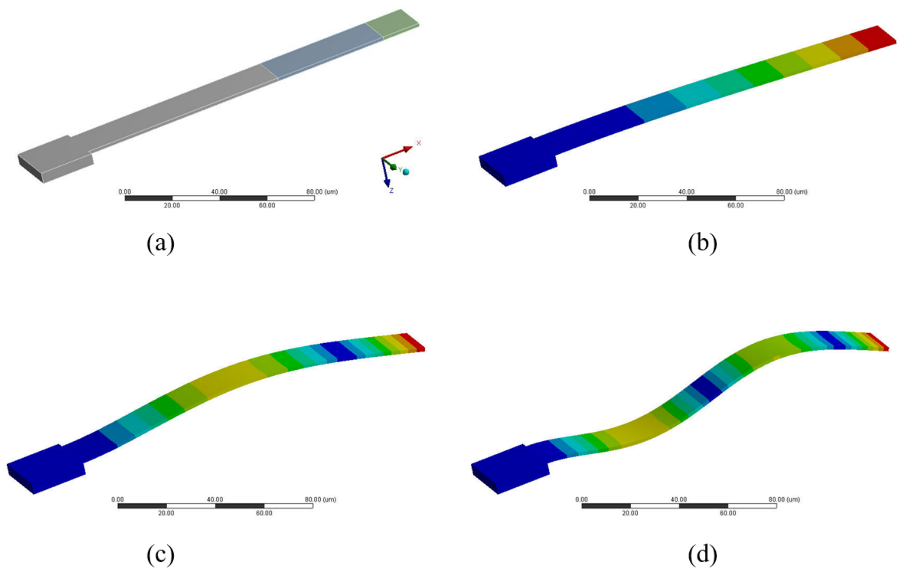


Fig. 8. The modal analysis of a polysilicon microcantilever. (a)geometrical model emphasizing electrode position; (b) the 1st mode; (c) the 2nd mode; (d) the 3rd mode of vibration with the acting electrode in position 1 (color online).

The computed resonant frequency of microcantilever for different positions of the acting electrode is presented in Table 1. These results are in good agreement with experimental data for resonant frequency, as presented in previous paragraph.

Table 1. Resonant frequency for different electrode positions and different oscillation modes obtained by FEM

Electrode position	Resonant frequency [kHz] / Oscillation mode		
	I	*II*	*III*
Position 0	78.32	490.50	1373.1
Position 1	78.43	491.03	1354.8
Position 2	78.40	490.90	1353.9
Position 3	78.40	491.02	1352.8
Position 4	78.37	491.02	1351.8

3. Results and discussions

The effect of the electrode position on the dynamic response of a resonator cantilever is experimentally investigated under the same applied voltage. The resonant frequency is not significantly influenced by the electrode position either in the presence of air or in vacuum, aspect confirmed also by the numerical results.

Figure 9 presents the electrode position effect on the frequency response of investigated resonator. The biggest effect is observed in the first bending mode where the resonant frequency increases with 9.5% in air and 9.7% in vacuum if the position of the applied force is moved from the beam free-end toward to the anchor.

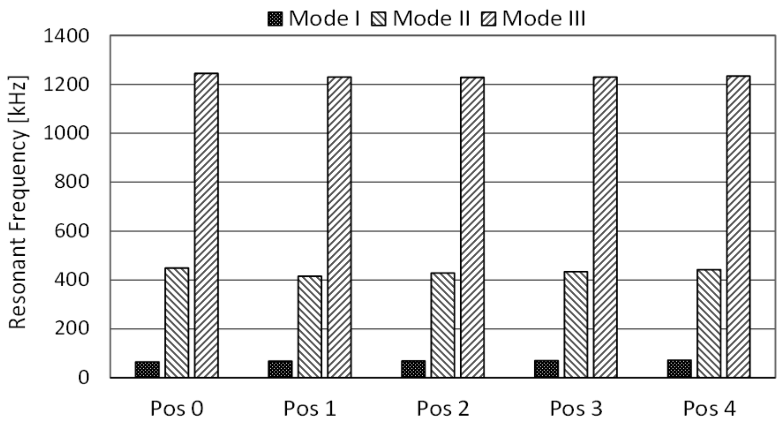
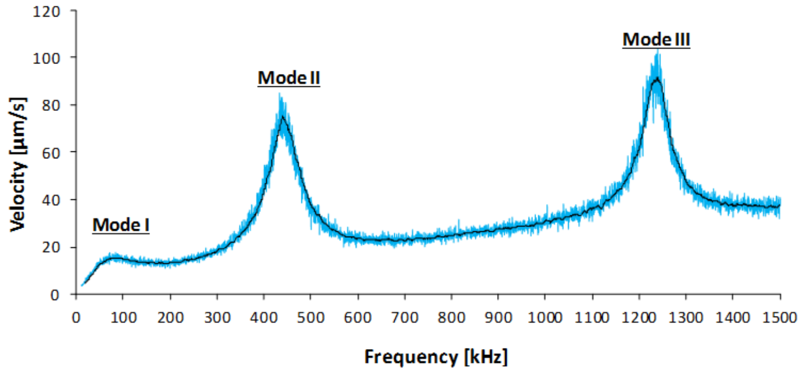
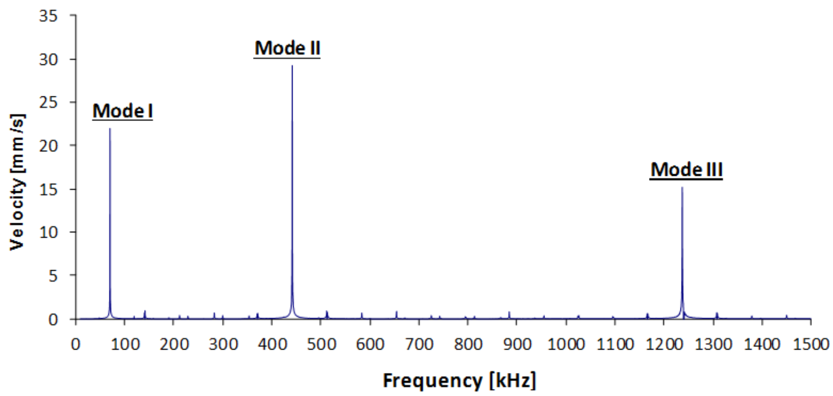


Fig. 9. Resonant frequency as a function of the electrode possition for different oscillation modes in air.



(a)



(b)

Fig. 10. Velocity of oscillations for different bending modes of the cantilever with the acting electrode close to the anchor in air (a) and vacuum (b).

The velocity of oscillation of the free-end of beam is modified if the acting electrode is moved from the cantilever free-end toward anchor. Figure 10 presents the velocity of oscillations in the case of cantilever with the acting electrode close to the anchor in air (Fig. 10a) and vacuum (Fig. 10b). In vacuum, the velocity of oscillations decreases with 38% for the first bending mode, 46% in the case of the second bending mode and with 40% for the third bending mode if the acting electrode is applied close to anchor (position 4, Fig. 4). The main loss of energy in vacuum is given by the internal dissipation energy that depends on the velocity of oscillation. As the velocity decreases, the Q- factor is improved. For the samples tested in ambient conditions, the velocity of oscillations decreases significantly if the acting electrode is moved from the cantilever free-end toward anchor under the same applied voltage. As the electrode position is changed, the pre-stress displacement of the cantilever decreases. Therefore, the gap between substrate and cantilever increases if the acting electrode is applied close to anchor that provide more dissipation energy during oscillation in ambient conditions. Comparatively with the case where the acting electrode is applied at the cantilever free-end (Fig. 7), the velocity of oscillations decreases with 92% for the first bending mode, 64% for second mode and 7.5% for the third mode for the resonator with the acting electrode positioned close to the anchor.

By using the experimentally obtained frequency response curves, the Q- factor is determined for different oscillation modes in air and vacuum. The experimental Q- factor of resonator for different position of the acting electrode and different oscillation modes tested in ambient conditions is presented in Table 2 and the results obtained in vacuum are included in Table 2.

Table 2. Q- factor for different electrode positions and different oscillation modes in ambient condition

Electrode position	**Oscillation mode**		
	I	*II*	*III*
Position 0	0.71	5.44	11.78
Position 1	0.69	8.83	18.16
Position 2	0.62	8.71	19.50
Position 3	0.56	8.57	21.98
Position 4	0.53	8.08	23.09

Table 3. Q- factor for different electrode positions and different oscillation modes in vacuum (7×10^{-4} mbar)

Electrode position	**Oscillation mode**		
	I	*II*	*III*
Position 0	32.00	739.28	1540.75
Position 1	41.26	860.74	1725.98
Position 2	50.19	850.25	1954.26
Position 3	58.95	1198.08	2125.04
Position 4	64.48	1237.81	2335.49

4. Conclusions

The reliability of MEMS devices with a microcantilever as an oscillating structure depends on the operating condition and the position of the applied load. The dynamic response of a microresonator can be modified only by changing the position of the acting electrode. In this paper, the position of the lower electrode is moved from the beam free-end toward to the beam anchor and the change in the dynamic response is monitored for the same impute voltage. The resonant frequency of a microcantilever is less influenced by the position of the acting electrode as well as the operating conditions. A significant influence of the testing conditions is observed on the velocity of oscillations which are increasing if the resonator operates in vacuum instead of the ambient conditions. The effect of the electrode position on velocity is determined. For the same input voltage the velocity of oscillations decreases if the acting electrode is moved from the cantilever free end toward anchor.

The quality factor that describes the quality of a microresonator has been estimated for different oscillation modes. In vacuum the damping effect given by air depends on the vacuum pressure. The Q- factor is significantly increases if the sample is oscillated in the 2nd or 3rd bending modes comparatively with the first one. In order, to improve the Q- factor and to decrease the loss of energy in applications where vibrating resonator is involved, excitation of structure in a higher bending mode can provide high accuracy in response and a long lifetime of device.

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