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A Novel Multiplicative Fuzzy Regression Function with A Multiplicative Fuzzy Clustering Algorithm

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Abstract. Possible structures of the system which is composed of various input and output variables are described by Fuzzy System Modeling(FSM). Traditional FSM approaches such as fuzzy rule-based systems and fuzzy regression functions have high ability to ensure approximating the real-world systems. *Fuzzy Functions with Least squares Estimation(FF-LSE)* is proposed by Turksen [1] for development of fuzzy system models. In the FF-LSE method, Improved Fuzzy Clustering(IFC) is used to find membership values in regression and classification type datasets, separately. In this study, we propose a novel FSM approach, namely *Multiplicative Fuzzy Regression Function(MFRF)*, which is constructed based on a new *Multiplicative Fuzzy Clustering(MFC)* algorithm. In the MFC algorithm, membership values are initially computed by Fuzzy c-Means Clustering(FCM) algorithm, then additional transformations of the membership values are used to generate multiplicative fuzzy functions(MFFs) for each cluster. The additional transformations of the membership values together with input variables are used by the Least Squares Estimation to form Multiplicative Fuzzy Regression Functions for each cluster identified by Multiplicative Fuzzy Clustering. Computational complexity of the proposed MFRF method is discussed and its performance is examined using several experiments on Concrete Compressive Strength dataset. Performance of the proposed MFRF is compared to FF-LSE and classical LSE approaches.

Key-words: Fuzzy Systems; Databases and Information Systems; Data Mining and Analysis.

1. Introduction

Cluster analysis is used for clustering a dataset into groups of similar individuals. Conventional (crisp) clustering methods put each point of the dataset to exactly one cluster [2]. Since the introduction of fuzzy sets by Zadeh [3], various fuzzy set-based approaches have been developed to model systems with uncertainties, such as fuzzy clustering methods. Fuzzy c-Means

(FCM) algorithm proposed by Bezdek [4] is well-known and commonly used as a fuzzy clustering method. FCM algorithm requires the predefined number of clusters, which is not known in advance. Variations of FCM algorithm were developed by several authors: Hathaway and Bezdek [5] proposed Fuzzy c-regression model (FCRM) clustering algorithm which can be used to fit switching regression models to certain types of mixed data; Hoppner and Klawon [6] combined FCM with FCRM algorithms in one schema; Pedrycz [7] discussed the fuzzy clustering with knowledge-based supervision and contrasted with the pure (data-driven) version of fuzzy clustering; Celikyilmaz and Turksen [8] proposed Improved Fuzzy Clustering algorithm. There are many different validity indices to determine the number of clusters. Some of these are Partition Coefficient and Partition Entropy of Bezdek [4, 9, 10], index of Fukuyama and Sugeno [11], index of Xie and Beni [12], index of Kim and Ramakrishna [13], index of Kung and Lin [14], index of Pakhira, Bandyopadhyay and Maulik [15], index of Celikyilmaz and Turksen [16] and index of Baskir and Turksen [17].

Fuzzy System Modeling (FSM) identifies possible structures of the given system domain that consists in various input and output variables. With application of FSM, one can build multi-structured models that represent relationships between these variables [18]. A fuzzy system modelling approach based on improved fuzzy functions is proposed by Celikyilmaz and Turksen [19] to model systems with continuous output variable. This approach introduces three features: *(i)* an Improved Fuzzy Clustering (IFC) algorithm, *(ii)* a new structure identification algorithm, and *(iii)* a nonparametric inference engine. The IFC algorithm yields simultaneous estimates of parameters of c-regression models, together with fuzzy c-partitioning of the data, to calculate improved membership values. Celikyilmaz and Turksen [16] introduced two cluster validity indices for improved fuzzy clustering (IFC) to be used to find patterns in regression and classification type datasets, separately. Turksen [1] proposed *Fuzzy Functions* to be determined by the Least Squares Estimation (LSE) technique for the development of fuzzy system models. These functions called as *Fuzzy Functions with Least Squares Errors (FF-LSE)* were introduced as an alternative to fuzzy rule-based approaches. In the method, a fuzzy clustering algorithm, such as Fuzzy c-Means Clustering (FCM) or its variations, is applied to obtain membership values of input vectors. Then, the membership values together with scalar input variables were used by the LSE to determine Fuzzy Functions for each cluster. Celikyilmaz and Turksen [19] presented a type-2 fuzzy function system for uncertainty modeling using evolutionary algorithms (ET2FF). The ET2FF algorithm based on the improved fuzzy clustering (IFC) algorithm and Genetic Algorithm (GA) is used to optimize the clustering parameters for identifying the uncertainty interval of the membership functions as well as the structure of the local functions. Golec *et al.* [20] examined the long-run relationship between the Shanghai index and CRB commodity index. The Fuzzy Functions with FCM and IFC, Adaptive Network Fuzzy Inference System (ANFIS), GENFIS, classical LSE and three versions of support vector regression (SVR) were applied to increase the performances of *RMSE*, R^2 and adjusted R^2. Ozkan and Turksen [21] proposed a validation criterion called ϵ-stable cluster validity index, which can be classified as a stability type validation. In particular, the stability of the clusters is assessed by means of the uncertainty associated with the level of fuzziness in FCM clustering algorithms. Baskir and Turksen [17] proposed an enhanced fuzzy clustering algorithm related to α-cut interval descriptions of fuzzy numbers and a cluster validity index, which is occurred by α-cut intervals and adding two ad-hoc functions in the compactness and separability measures. Nagwani and Deo [22] aimed to demonstrate that clustering algorithm along with regression ensures less prediction errors for estimating the concrete compressive strength. The proposed technique consists of two stages: *(i)* clustering algorithms, i.e. FCM and k-means were used to group the similar characteristics of concrete data, *(ii)* regression techniques were applied over these clusters (groups) to predict the compressive strength from individual clusters.

In this study, we introduce a new Multiplicative Fuzzy Clustering (MFC) algorithm and propose a novel Multiplicative Fuzzy Regression Functions (MFRF) based on the MFC algorithm to represent fuzzy system modeling. The proposed MFRF is applied to the Concrete Compressive Strength data set and the results show that the MFRF approach is comparable, although not superior to the FF-LSE approach in terms of performance evaluation.

This study is organized as follows; In Section 2, we described a new multiplicative fuzzy clustering (MFC) algorithm based on multiplicative fuzzy functions. In Section 3, we proposed a novel multiplicative fuzzy regression functions (MFRF) algorithm based on MFC algorithm. In Section 4, we showed the results of MFRF, FF-LSE, and LSE on the Concrete Comprehensive Strength dataset taken from UCI Machine Learning Repository. Finally, conclusions were drawn in Section 5.

2. Proposed Multiplicative Fuzzy Clustering Functions

Fuzzy clustering is the partitioning of a data set into fuzzy subsets or clusters based on the similarities between the data [23]. One of the well-known fuzzy clustering methods introduced by Bezdek [4] is Fuzzy c-Means (FCM) algorithm. The FCM algorithm is used in fuzzy system models to find the membership values, which are assumed to represent optimum partitions of the given dataset. Hathaway and Bezdek [5] presented Fuzzy c-Regression Models (FCRM) which can be used to fit switching regression models to certain types of mixed data. In the FCRM method, minimization of particular objective functions in the family yields simultaneous estimates for the parameters of c regression models, together with a fuzzy c-partitioning of the data. Improved Fuzzy Clustering (IFC) algorithm is a hybrid-type clustering method which combines FCM algorithm [4] and FCRM algorithm [5] within one clustering schema and it utilizes fuzzy functions. IFC algorithm was proposed by Celikyilmaz and Turksen [8] for regression and classification type domains and its variation. In the IFC algorithm, the membership values are used as additional input variables to estimate the parameters of the regression models for each cluster. The optimization approach of the IFC algorithm not only searches the best partition of the data but also aims to increase the predictive power of the membership values to model the output variable with Fuzzy Function [8, 19, 24, 25].

In this study, we propose a new *Multiplicative Fuzzy Clustering (MFC)* algorithm in the light of IFC algorithm to find membership values by using multiplicative fuzzy functions (MFFs). Let denote each data point, where $x_k \in \Re^{nv}$ is any *nv*-dimensional *k*th input vector, $y_k \in \Re$ is its observed output, $\mu_{ik} = \mu_{ik}(x_k) \in [0, 1]$ indicate its membership value to the cluster $i = 1, 2, ..., c$. *c* denotes total number of clusters, *n* is the total number of data vectors and $m > 1$ is the degree of fuzziness parameter. In the proposed MFC algorithm, objective function J_{MFC} is minimized to optimize the membership values as follows:

$$
\begin{gathered}
Min\, J_{MFC} = \sum_{i=1}^{c} \sum_{k=1}^{n} (\mu_{ik})^m d_{ik}^2 + \sum_{i=1}^{c} \sum_{k=1}^{n} (\mu_{ik})^m \left(y_k - f_i \left(\tau_{ik}, \hat{\beta}_i \right)_{MFC} \right)^2 \\
0 \le \mu_{ik} \le 1, \ \forall i, k; \\
\sum_{i=1}^{c} \mu_{ik} = 1, \ \forall k; \\
0 \le \sum_{k=1}^{n} \mu_{ik} \le n, \quad \forall i
\end{gathered} \tag{1}
$$

In Eq (1), the first term of the objective function is as same as the FCM algorithm which $d_{ik} = \|x_k - v_i\|$ represents the distance of each x_k to each cluster center v_i. The second term is the squared error between the observed output y_k and the Multiplicative Fuzzy Functions (MFFs) denoted as $f_i \left(\tau_{ik}, \hat{\beta}_i \right)_{MFC}$. The MFFs are built by using the membership values and/or their user-defined transformations as input vector excluding the original input vector. The multiplicative input matrix-τ_{ik} constituted to estimate these MFFs, can be given as follows:

$$
\tau_{ik} = \left[1 \quad \left(\mu_{ik} + \mu_{ik}^2 + ... + e^{\mu_{ik}^m} \right) \right]. \tag{2}
$$

An example of a MFF for cluster *i* is defined as:

$$
\begin{aligned}
\hat{y}_i = f_i \left(\tau_{ik}, \hat{\beta}_i \right)_{MFC} &= {\tau_{ik}}^T \hat{\beta} \\
&= \hat{\beta}_{i0} + \hat{\beta}_{i1} \left(\mu_{ik} + \mu_{ik}^2 + ... + e^{\mu_{ik}^m} \right)
\end{aligned} . \tag{3}
$$

In Eq (3), $\hat{y}_i$ is the estimated output value obtained from *i*th MFF and τ_{ik} is the multiplicative input matrix of cluster *i*. $\hat{\beta}_i = \left[\hat{\beta}_{i0}, \ \hat{\beta}_{i1} \right]^T$ represents regression parameter vector of the *i*th MFF using the least squares estimation method.

The cluster centers and related membership values obtained from the MFC algorithm, which is the solution of the constrained optimization problem in Eq (1), are given in Eqs. (4) and (5), respectively:

$$
\mu_{ik,t}^{MFC} = \left[\sum_{j=1}^{c} \left(\frac{\|x_k - v_{i,t-1}\|^2 + \left(y_k - f_i \left(\tau_{ik,t-1}, \hat{\beta}_i \right)_{MFC} \right)^2}{\|x_k - v_{j,t-1}\|^2 + \left(y_k - f_j \left(\tau_{jk,t-1}, \hat{\beta}_i \right)_{MFC} \right)^2} \right)^{1/m-1} \right]^{-1} . \tag{4}
$$

$$
v_{i,t}^{MFC} = \frac{\sum_{k=1}^{n} \left(\mu_{ik,t}^{MFC} \right)^m x_k}{\sum_{k=1}^{n} \left(\mu_{ik,t}^{MFC} \right)^m}. \tag{5}
$$

The algorithm of the Multiplicative Fuzzy Clustering (MFC) is given as follows:
Step 1. Choose parameters $(c, m, iter, \varepsilon)$.
Step 2. Find the initial membership values μ_{ik} using the FCM algorithm.
Step 3. ITERATE
For $t = 1$ to *iter*

- Construct the multiplicative input matrix-$\tau_{ik,t}$ for cluster *i* such as in Eq (2)
- Approximate the multiplicative fuzzy function-$f_i\left(\tau_{ik,t},\hat{\beta}_i\right)_{MFC}$ for cluster *i* such as in Eq (3)
- Update the membership values by using Eq (4)
- Update the cluster centers via Eq (5)

If $t = iter$ and $\left(J_{MFC}^{(t)} - J_{MFC}^{(t-1)}\right) < \varepsilon$ then STOP

Next *t*

3. Proposed Multiplicative Fuzzy Regression Functions

The Fuzzy Functions with Least Squares Errors (FF-LSE) proposed by Turksen [1] was introduced to be determined by the least squares estimation (LSE) technique for the development of fuzzy system models. The FF-LSE's are structurally different from *Fuzzy Rule-Based* approaches presented by Zadeh [26], Takagi and Sugeno [27]. They are also different from *Fuzzy Regression* models introduced by Tanaka *et al.* [28] and Hathaway and Bezdek [5]. In the FF-LSE approach, membership values and/or their user-defined transformations are considered as input vectors in addition to original input matrix. For more details on the FF-LSE, the readers can find them in Turksen [1].

Now, we introduce a novel *Multiplicative Fuzzy Regression Function (MFRF)*, an alternate approach to the FF-LSE for regression type data. In the proposed MFRF approach, the membership values and/or their user-defined transformations are considered as multipliers of the original input matrix-$X \in \Re^{n\times nv}$. Then, a new multiplicative interim input matrix-$\Phi_i\left(x\mu_i\right)$, is formed in order to obtain the best representation of a system behavior. $\Phi_i\left(x\mu_i\right)$ which can be denoted as $X_i^* \in \Re^{n\times(nv+1)}$, is constructed for each of the clusters. Considering the membership values μ_{ik} obtained from the MFC algorithm for cluster *i*, several examples of X_i^* could be given as follows:

$$X_i^* = \left[1 \;\; \mu_{ik}X\right]. \tag{6}$$

$$X_i^* = \left[1 \;\; \left(\mu_{ik} + \mu_{ik}^2\right)X\right]. \tag{7}$$

$$X_i^* = \left[1 \;\; \mu_{ik}^m X\right]. \tag{8}$$

$$X_i^* = \left[1 \;\; e^{\mu_{ik}}X\right]. \tag{9}$$

$$X_i^* = \left[1 \;\; \left(\mu_{ik} + \log\left(\mu_{ik}\right)\right)X\right]. \tag{10}$$

$$X_i^* = \left[1 \;\; \left(\mu_{ik} + \mu_{ik}^2 + ... + e^{\mu_{ik}^m}\right)X\right]. \tag{11}$$

Thus, the regression parameter vector-$\beta_i{}^* = \left[\beta_{i,0}^*, \beta_{i,1}^*, ..., \beta_{i,nv}^*\right]^T$ of the MFF for cluster *i*, is estimated by the LSE, as follows:

$$\beta_i^* = \left(X_i^{*T}X_i^*\right)^{-1}X_i^{*T}Y. \tag{12}$$

The estimated output-$\hat{y}_i$ for cluster *i* can be obtained as,

$$\hat{y}_i = f_i\left(\Phi_i\left(x\mu_i\right), \hat{\beta}_i\right) = \beta^*_{i,0}+\beta^*_{i,1}(\mu_i+\mu_i^2+...+e^{\mu_i m})X_1+...+\beta^*_{i,nv}(\mu_i+\mu_i^2+...+e^{\mu_i m})X_{nv}. \quad (13)$$

The single output is computed by weighting the estimated outputs from each cluster with their corresponding membership values, as follows:

$$y^* = \frac{\sum_{i=1}^{c^*}\left(\mu_i + {\mu_i}^2 + ... + e^{\mu_i m}\right)\hat{Y}_i}{\sum_{i=1}^{c^*}\left(\mu_i + {\mu_i}^2 + ... + e^{\mu_i m}\right)} \quad (14)$$

The algorithm of the proposed Multiplicative Fuzzy Regression Functions (MFRF) consists of two stages. The first stage includes determination of the membership values according to the proposed MFC algorithm. The second stage conmprises computation of the estimated outputs. The algorithm of the MFRF illustrated in Figure 1, is given as follows:

Stage 1

Step 1. Choose parameters $(c, m, iter, \varepsilon)$.

Step 2. Compute the membership values-μ_{ik} and the cluster centers-v_i for each cluster

i by using the MFC algorithm.

Stage 2

Step 3. ITERATE

For $t = 1$ to *iter*

- Construct the multiplicative interim input matrix-$\Phi_i\left(x\mu_i\right)$ for each cluster *i*, as in Eqs (6)-(11)

- Estimate the regression parameters-β_i^* via Eq (12) for cluster *i*

- Infer the output variable value of the data vectors using the fuzzy functions parameters by using Eq (13)

- Find the single output variable value by weighting the estimated output variable values as in Eq (14)

If $t = iter$ and $(J_t - J_{t-1}) < \varepsilon$ then STOP.

Next *t*

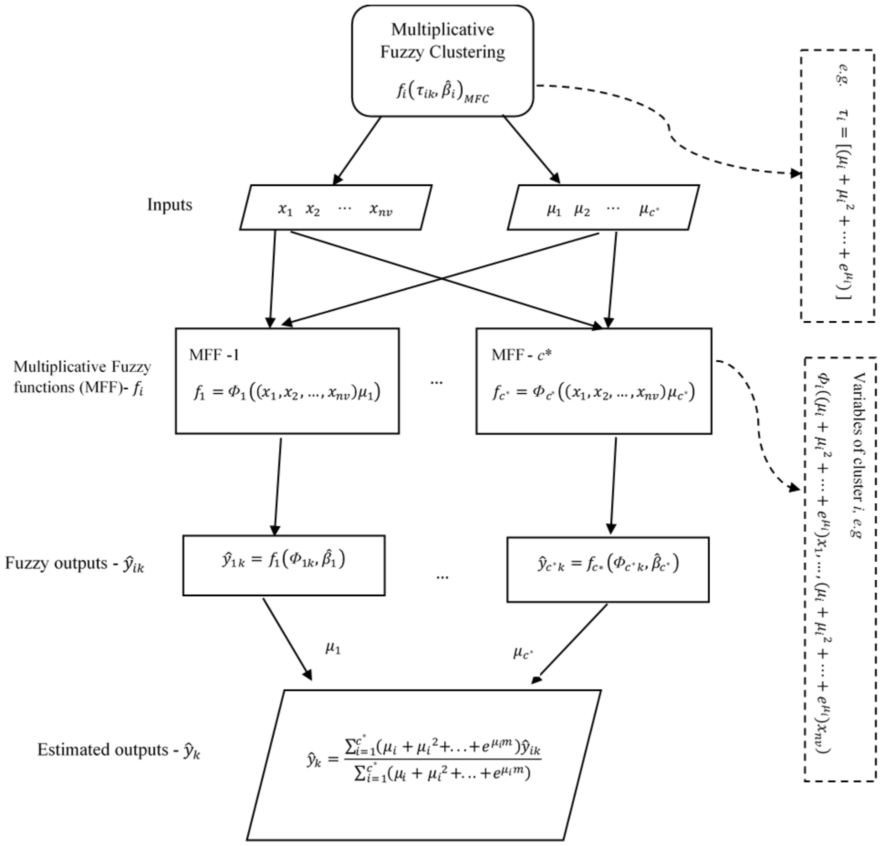

Fig. 1. Structure of the Multiplicative Fuzzy Regression Function.

4. Numerical Examples

In this section, we present an experimental analysis conducted using the proposed Multiplicative Fuzzy Regression Function (MFRF) approach. Description of the regression type dataset of *Compressive Concrete Strength*, where eight inputs and one output, from UCI Machine Learning Repository, are shown in Table 1 [[29], [30]]. We compare the MFRF results with the FF-LSE and LSE.

For Benchmark datasets, approximately 80% of observations (n=824) from each dataset are randomly selected for training and 20% of observations (n=206) are used for testing the model performance. Experiments are repeated for ten random subsets of training and testing

Table 1. Characteristics of Concrete Compressive Strength data set.

Components	Measurement	Description	Min	Max	Mean ± Std. deviation
Cement	kg/m3	Input	102.0	540.0	281.16 ± 104.50
Blast Furnace Slag	kg/m3	Input	0.0	359.4	73.89 ± 86.27
Flay Ash	kg/m3	Input	0.0	200.1	54.18 ± 63.99
Water	kg/m3	Input	121.8	247.0	181.56 ± 21.35
Superplasticizer	kg/m3	Input	0.0	32.2	6.20 ± 5.97
Course	kg/m3	Input	801.0	1145.0	972.91 ± 77.75
Fine Aggregate	kg/m3	Input	594.0	992.6	773.57 ± 80.17
Age	Day (1 365)	Input	1.0	365.0	45.66 ± 63.16
Concrete Compressive Strength	MPa	Output	2.3	82.6	35.81 ± 16.70

datasets of the above sizes. The R^2 and $RMSE$ values are used to measure performance of the models over ten experiments for considered six models (M1, M2, M3, M4, M5, M6) according to the different fuzzy clustering parameters, i.e. $c = (2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10)$ and $m = (1.4,\ 1.6,\ 1.8,\ 2.0,\ 2.2,\ 2.4,\ 2.6)$. The proposed MFRF approach attempts to find a good representation of a system behavior. The membership values computed by the MFC algorithm and their user-defined transformations are considered as a multiplier of the original inputs in order to estimate regression models in each cluster. However, the membership values are used as additional inputs to the original inputs in the FF-LSE. Therefore, the proposed MFRF approach is an alternative to the FF-LSE approach. In the experiments, the input matrices used in the IFC, MFC, FF-LSE, and MFRF for six different models are shown in Table 2.

Table 2. The input matrices of the IFC, FF-LSE, MFC, and MFRF for considered six models.

Model	IFC- $\tau_i\left(\mu_i\right)_{IFC}$	FF-LSE - $\Phi_i\left(x, \mu_i\right)$
M1	$X' = [1,\ \mu_{ik}]$	$X^* = [1,\ \mu_{ik},\ X]$
M2	$X' = \left[1,\ \mu_{ik}, \mu_{ik}^2\right]$	$X^* = \left[1,\ \mu_{ik}, \mu_{ik}^2,\ X\right]$
M3	$X' = \left[1,\ \mu_{ik}^m\right]$	$X^* = \left[1,\ \mu_{ik}^m,\ X\right]$
M4	$X' = [1,\ e^{\mu_{ik}}]$	$X^* = [1,\ e^{\mu_{ik}},\ X]$
M5	$X' = \left[1,\ \mu_{ik}, \mu_{ik}^m, e^{\mu_{ik}^m}\right]$	$X^* = \left[1,\ \mu_{ik}, \mu_{ik}^m, e^{\mu_{ik}^m},\ X\right]$
M6	$X' = \left[1,\ \mu_{ik}, \mu_{ik}^m, e^{\mu_{ik}}, e^{\mu_{ik}^m}\right]$	$X^* = \left[1,\ \mu_{ik}, \mu_{ik}^m, e^{\mu_{ik}}, e^{\mu_{ik}^m},\ X\right]$
Model	**MFC-** $\tau_i\left(\mu_i\right)_{MFC}$	**MFRF-** $\Phi_i\left(x\mu_i\right)$
M1	$X' = [1,\ \mu_{ik}]$	$X^* = [1, \mu_{ik}X]$
M2	$X' = \left[1, \left(\mu_{ik} + \mu_{ik}^2\right)\right]$	$X^* = \left[1, \left(\mu_{ik} + \mu_{ik}^2\right) X\right]$
M3	$X' = \left[1,\ \mu_{ik}^m\right]$	$X^* = \left[1,\ \mu_{ik}^m X\right]$
M4	$X' = [1,\ e^{\mu_{ik}}]$	$X^* = [1,\ e^{\mu_{ik}} X]$
M5	$X' = \left[1,\ \left(\mu_{ik} + \mu_{ik}^m + e^{\mu_{ik}^m}\right)\right]$	$X^* = \left[1,\ \left(\mu_{ik} + \mu_{ik}^m + e^{\mu_{ik}^m}\right) X\right]$
M6	$X' = \left[1,\ \left(\mu_{ik}, \mu_{ik}^m, e^{\mu_{ik}}, e^{\mu_{ik}^m}\right)\right]$	$X^* = \left[1,\ \left(\mu_{ik} + \mu_{ik}^m + e^{\mu_{ik}} + e^{\mu_{ik}^m}\right) X\right]$

All computations are implemented using MATLAB on a PC with 2.50 GHz and 2-GB RAM and performed in three major steps: (*i*) the FCM, IFC, and proposed MFC algorithms are employed to compute the membership values, where the groups of similar concrete data are constituted, (*ii*) the regression based fuzzy function techniques, i.e. the MFRF and FF-LSE, are applied to estimate the concrete compressive strength, (*iii*) the coefficient of determination (R^2) and the root-mean square error (*RMSE*) values are recorded for 10 experiments from each training and testing data set to compare the performance of the MFRF, FF-LSE, and LSE.

The coefficient of determination (R^2) which evaluates how well the model fits the data, is calculated as follows:

$$R^2 = 1 - \frac{Sum\,of\,Squares\,Error}{Total\,Sum\,of\,Squares} = 1 - \frac{\sum_{k=1}^{n}(y_k - \hat{y}_k)^2}{\sum_{k=1}^{n}(y_k - \bar{y})^2}. \quad (15)$$

The root mean squared error (*RMSE*) which is useful to understand the deviation between observed and estimated output, is calculated by using:

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(y_k - \hat{y}_k)^2}. \quad (16)$$

In Eq (15) and Eq (16), y_k is the observed output and $\hat{y}_k$ is the estimated output for *k*th observation, and $\bar{y}$ indicates the mean of the y_k's.

Using the algorithm stated in Section 3, at first, the membership values are computed through the MFC algorithm. Then, the MFRF is applied for considered six models on each training/testing dataset. We compared the performance of the proposed MFRF with FF-LSE and LSE considering R^2 and *RMSE* values. In Table 3 , the numbers in each cell indicate the average of optimal R^2, *RMSE* and (*c*,m**) values over 10 experiments according to the considered models for training / testing datasets.

Table 3. The average of optimal R^2(%), *RMSE* and (*c*,m**) values for training and testing data sets.

			Training data sets				***Testing data sets***			
Approach	***Model***	***Model name***	***c****	***m****	***R²(%)***	***RMSE***	***c****	***m****	***R²(%)***	***RMSE***
LSE	-	-	-	-	63.23	10.18	-	-	65.62	8.80
FF-LSE	M1	FF-LSE -M1	6.10	1.86	65.62	9.79	6.50	1.88	68.24	8.14
	M2	FF-LSE -M2	***5.00***	***2.04***	***69.35***	***9.23***	6.50	1.88	70.18	7.85
	M3	FF-LSE -M3	***4.70***	***2.02***	***69.84***	***9.14***	6.40	1.78	70.15	7.86
	M4	FF-LSE -M4	***5.80***	***1.90***	***65.78***	***9.76***	4.60	2.00	68.09	8.16
	M5	FF-LSE -M5	***6.30***	***2.16***	***77.04***	***7.90***	7.30	2.30	72.73	7.50
	M6	FF-LSE -M6	***7.40***	***2.18***	***77.71***	***7.81***	***6.90***	***2.08***	***78.20***	***6.81***
MFRF	M1	MFRF-M1	***6.20***	***1.42***	***68.05***	***9.45***	***3.30***	***1.56***	***73.69***	***7.40***
	M2	MFRF-M2	5.30	1.48	68.17	9.43	***3.20***	***1.44***	***75.06***	***7.19***
	M3	MFRF-M3	5.90	1.46	68.21	9.42	***4.00***	***1.44***	***74.87***	***7.21***
	M4	MFRF-M4	2.20	1.42	64.83	9.89	***3.20***	***1.60***	***69.00***	***8.02***
	M5	MFRF-M5	2.70	1.50	66.61	9.66	***2.80***	***1.46***	***71.99***	***7.60***
	M6	MFRF-M6	2.60	1.50	66.08	9.73	2.60	1.44	70.75	7.76

*c**: average optimal cluster number; *m**:average optimal fuzziness degree; *RMSE* and R^2 represent average of optimal values calculated from 10 experiment for training/testing data sets; Bold numbers indicate optimal results

According to Table 3, the MFRF approach is more appropriate than the FF-LSE approach for only model M1 when the training data sets are considered. On the other hand, the MFRF approach is more suitable than the FF-LSE for the models M2, M3, M4, M5 and M6 when the testing datasets are considered. Both the MFRF and FF-LSE have better solutions than the LSE for training / testing datasets with respect to the considered six models.

In this study, we also obtained the average of optimal R^2, *RMSE* and fuzziness degree (m) values with respect to different cluster numbers (c) computed by the MFRF and FF-LSE. The results obtained for the six models over 10 experiments for training / testing datasets are shown in Table 4 and Table 5, respectively. Figure 2 depicts $RMSE$ values versus cluster numbers (c) and Figure 3 illustrates R^2 values versus cluster numbers (c) calculated from six models over 10 experiments for training / testing data sets, respectively.

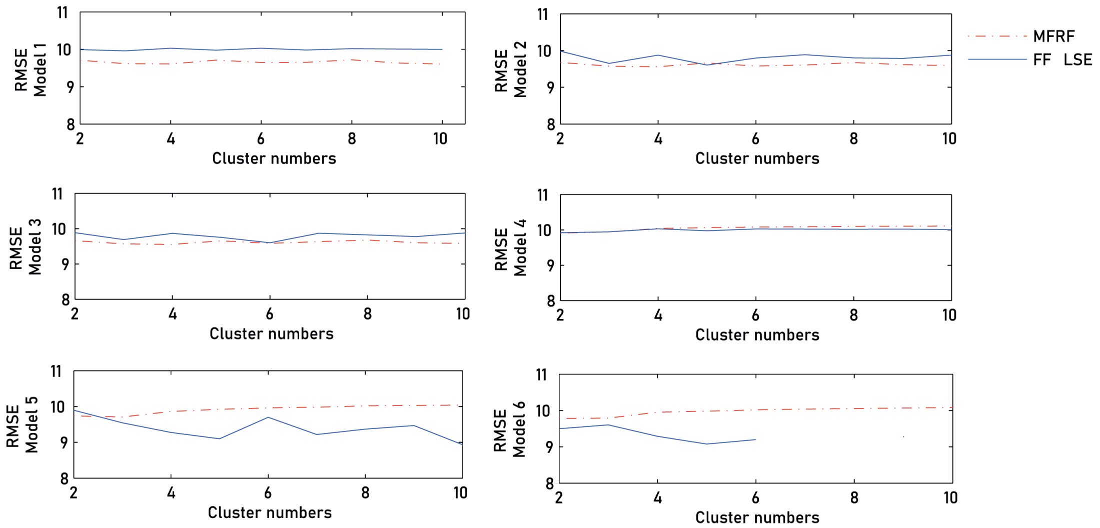

Fig. 2. *RMSE* values *vs* cluster numbers (c) of each model for training datasets.

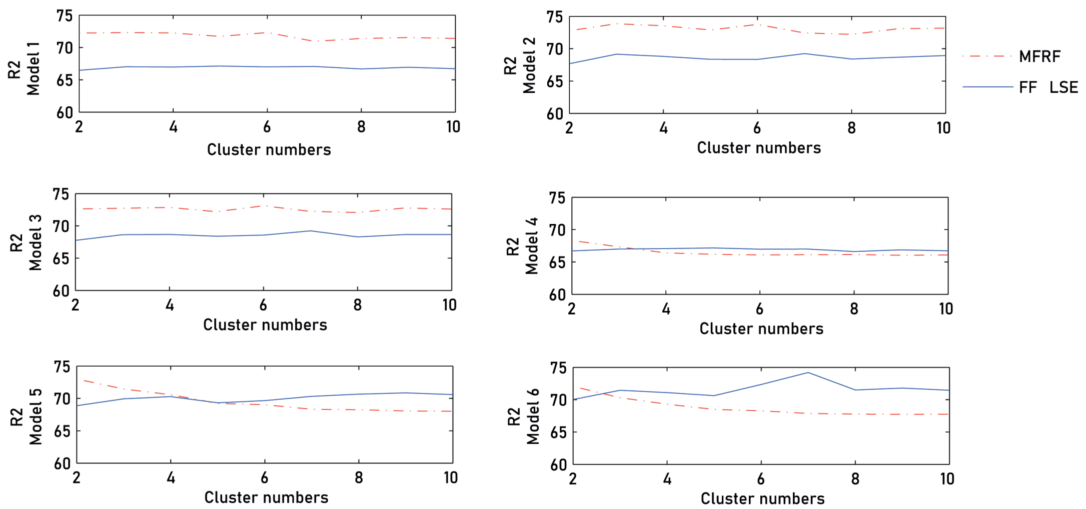

Fig. 3. R^2 values *vs* cluster numbers (c) of each model for testing datasets.

Table 4. The average of optimal R^2*(%)*, *RMSE* and *m* values with respect to different *c* values for each model for training datasets.

		FF-LSE				**MFRF**			
c	**Model**	**Model name**	***m***	**R^2*(%)***	***RMSE***	**Model name**	***m***	**R^2*(%)***	***RMSE***
2	M1	FF-LSE -M1	2,36	64,18	9,9901	MFRF-M1	***1,54***	***66,25***	***9,7050***
	M2	FF-LSE -M2	2,32	64,22	9,9847	MFRF-M2	***1,62***	***66,47***	***9,6746***
	M3	FF-LSE -M3	2,26	64,92	9,8852	MFRF-M3	***1,60***	***66,59***	***9,6576***
	M4	FF-LSE -M4	2,10	64,66	9,9203	MFRF-M4	***1,46***	***64,81***	***9,9076***
	M5	FF-LSE -M5	2,18	64,87	9,8971	MFRF-M5	***1,58***	***66,03***	***9,7372***
	M6	FF-LSE -M6	***1,94***	***67,57***	***9,4970***	MFRF-M6	1,54	65,71	9,7821
3	M1	FF-LSE -M1	1,66	64,42	9,9575	MFRF-M1	***1,54***	***66,89***	***9,6161***
	M2	FF-LSE -M2	1,72	66,53	9,6504	MFRF-M2	***1,60***	***67,15***	***9,5756***
	M3	FF-LSE -M3	1,74	66,29	9,6877	MFRF-M3	***1,60***	***67,19***	***9,5695***
	M4	FF-LSE -M4	1,90	64,53	9,9447	MFRF-M4	***1,46***	***64,57***	***9,9434***
	M5	FF-LSE -M5	***1,72***	***67,3***	***9,5451***	MFRF-M5	1,52	66,22	9,7098
	M6	FF-LSE -M6	***1,78***	***66,92***	***9,6050***	MFRF-M6	1,52	65,66	9,7903
4	M1	FF-LSE -M1	1,96	63,94	10,0275	MFRF-M1	***1,48***	***66,9***	***9,6088***
	M2	FF-LSE -M2	2,10	65,03	9,8747	MFRF-M2	***1,52***	***67,25***	***9,5599***
	M3	FF-LSE -M3	2,06	65,14	9,8661	MFRF-M3	***1,52***	***67,29***	***9,5532***
	M4	FF-LSE -M4	***2,06***	***63,93***	***10,0296***	MFRF-M4	1,44	63,86	10,0399
	M5	FF-LSE -M5	***2,00***	***68,93***	***9,2780***	MFRF-M5	1,48	65,14	9,8618
	M6	FF-LSE -M6	***1,90***	***68,81***	***9,2875***	MFRF-M6	1,40	64,46	9,9553
5	M1	FF-LSE -M1	1,80	64,33	9,9742	MFRF-M1	***1,56***	***66,24***	***9,7100***
	M2	FF-LSE -M2	***2,04***	***66,74***	***9,6057***	MFRF-M2	1,62	66,55	9,6622
	M3	FF-LSE -M3	2,00	65,81	9,7539	MFRF-M3	***1,60***	***66,61***	***9,6544***
	M4	FF-LSE -M4	***1,84***	***64,33***	***9,9749***	MFRF-M4	1,42	63,72	10,0607
	M5	FF-LSE -M5	***2,08***	***70,29***	***9,0980***	MFRF-M5	1,42	64,72	9,9233
	M6	FF-LSE -M6	***2,08***	***69,97***	***9,0777***	MFRF-M6	1,42	64,29	9,9822
6	M1	FF-LSE -M1	1,76	63,93	10,0289	MFRF-M1	***1,56***	***66,65***	***9,6488***
	M2	FF-LSE -M2	2,00	65,49	9,7975	MFRF-M2	***1,56***	***67,13***	***9,5794***
	M3	FF-LSE -M3	2,00	66,61	9,6015	MFRF-M3	***1,56***	***67,1***	***9,5838***
	M4	FF-LSE -M4	***1,94***	***63,95***	***10,0248***	MFRF-M4	1,44	63,56	10,0818
	M5	FF-LSE -M5	***1,98***	***66,24***	***9,7001***	MFRF-M5	1,46	64,42	9,9618
	M6	FF-LSE -M6	***1,86***	***69,10***	***9,1988***	MFRF-M6	1,48	64,00	10,0197
7	M1	FF-LSE -M1	1,74	64,31	9,9805	MFRF-M1	***1,52***	***66,66***	***9,6509***
	M2	FF-LSE -M2	1,84	64,93	9,8871	MFRF-M2	***1,60***	***66,98***	***9,6041***
	M3	FF-LSE -M3	2,02	65,04	9,8687	MFRF-M3	***1,60***	***66,78***	***9,6320***
	M4	FF-LSE -M4	***1,68***	***63,98***	***10,0207***	MFRF-M4	1,42	63,51	10,0892
	M5	FF-LSE -M5	***2,06***	***69,24***	***9,2181***	MFRF-M5	1,40	64,29	9,9824
	M6	FF-LSE -M6	1,96	NaN	NaN	MFRF-M6	***1,40***	***63,9***	***10,0357***
8	M1	FF-LSE -M1	1,9	64.00	10,0181	MFRF-M1	***1,58***	***66,18***	***9,7171***
	M2	FF-LSE -M2	1,98	65,53	9,7987	MFRF-M2	***1,60***	***66,49***	***9,6724***
	M3	FF-LSE -M3	2,04	65,37	9,8230	MFRF-M3	***1,60***	***66,45***	***9,6786***
	M4	FF-LSE -M4	***1,84***	***64,01***	***10,016***	MFRF-M4	1,46	63,43	10,1003
	M5	FF-LSE -M5	***2,06***	***68,25***	***9,3674***	MFRF-M5	1,44	64,00	10,0212
	M6	FF-LSE -M6	1,76	NaN	NaN	MFRF-M6	***1,44***	63,73	***10,0579***
9	M1	FF-LSE -M1	1,68	64,09	10,0059	MFRF-M1	***1,50***	***66,75***	***9,6331***
	M2	FF-LSE -M2	1,86	65,73	9,7860	MFRF-M2	***1,52***	***66,87***	***9,6148***
	M3	FF-LSE -M3	1,94	65,80	9,7751	MFRF-M3	***1,52***	***66,97***	***9,6004***
	M4	FF-LSE -M4	***1,76***	***63,97***	***10,0233***	MFRF-M4	1,44	63,41	10,1021
	M5	FF-LSE -M5	***1,80***	***67,77***	***9,4684***	MFRF-M5	1,46	63,94	10,0291
	M6	FF-LSE -M6	***1,98***	***68,80***	***9,2773***	MFRF-M6	1,44	63,66	10,0675
10	M1	FF-LSE -M1	1,78	64,16	9,9995	MFRF-M1	***1,52***	***66,92***	***9,6038***
	M2	FF-LSE -M2	1,98	65,01	9,8754	MFRF-M2	***1,52***	***67,02***	***9,5910***
	M3	FF-LSE -M3	1,88	64,98	9,8761	MFRF-M3	***1,52***	***67,10***	***9,5796***
	M4	FF-LSE -M4	***1,90***	***64,10***	***10,0076***	MFRF-M4	1,48	63,38	10,1069
	M5	FF-LSE -M5	***2,04***	***70,47***	***8,9322***	MFRF-M5	1,44	63,82	10,0443
	M6	FF-LSE -M6	2,02	NaN	NaN	MFRF-M6	***1,44***	***63,57***	***10,0795***

m, *RMSE* and R^2 represent average values obtained from 10 experiments; Bold numbers indicate optimal results for each cluster

Table 5. The average of optimal R^2*(%)*, *RMSE* and *m* values with respect to different *c* values for each model for testing datasets.

		FF-LSE				**MFRF**			
c	**Model**	**Model name**	***m****	***R*²*(%)***	***RMSE***	**Model name**	***m****	***R*²*(%)***	***RMSE***
2	M1	FF-LSE -M1	2,14	66,47	6,1401	MFRF-M1	***1,62***	***72,22***	***7,7229***
	M2	FF-LSE -M2	2,08	67,68	6,1131	MFRF-M2	***1,52***	***72,78***	***7,6124***
	M3	FF-LSE -M3	2,06	67,75	6,0838	MFRF-M3	***1,62***	***72,63***	***7,6218***
	M4	FF-LSE -M4	1,88	66,70	6,0951	MFRF-M4	***1,56***	***68,33***	***8,2379***
	M5	FF-LSE -M5	2,08	68,88	6,0747	MFRF-M5	***1,58***	***73,01***	***7,7600***
	M6	FF-LSE -M6	1,88	70,03	5,8117	MFRF-M6	***1,56***	***72,05***	***7,9061***
3	M1	FF-LSE -M1	2,16	67,01	6,0523	MFRF-M1	***1,58***	***72,31***	***7,7346***
	M2	FF-LSE -M2	2,10	69,16	5,8452	MFRF-M2	***1,48***	***73,86***	***7,4963***
	M3	FF-LSE -M3	2,16	68,63	5,9509	MFRF-M3	***1,48***	***72,73***	***7,6081***
	M4	FF-LSE -M4	2,20	66,99	6,0695	MFRF-M4	***1,58***	***67,34***	***8,3926***
	M5	FF-LSE -M5	2,14	69,94	5,9414	MFRF-M5	***1,47***	***71,45***	***7,9900***
	M6	FF-LSE -M6	***2,04***	***71,46***	***5,7461***	MFRF-M6	1,51	70,3	8,1619
4	M1	FF-LSE -M1	2,68	66,97	6,0258	MFRF-M1	***1,54***	***72,24***	***7,7354***
	M2	FF-LSE -M2	2,56	68,81	5,9213	MFRF-M2	***1,44***	***73,54***	***7,5287***
	M3	FF-LSE -M3	2,42	68,68	5,9344	MFRF-M3	***1,56***	***72,86***	***7,5910***
	M4	FF-LSE -M4	***2,62***	***67,06***	***6,1159***	MFRF-M4	1,52	66,4	8,5088
	M5	FF-LSE -M5	2,58	70,29	5,9825	MFRF-M5	***1,44***	***70,58***	***8,1300***
	M6	FF-LSE -M6	***2,40***	***71,08***	***5,8393***	MFRF-M6	1,44	69,31	8,3065
5	M1	FF-LSE -M1	2,90	67,13	6,0513	MFRF-M1	***1,62***	***71,71***	***7,8236***
	M2	FF-LSE -M2	2,74	68,37	5,9931	MFRF-M2	***1,54***	***72,90***	***7,6352***
	M3	FF-LSE -M3	2,68	68,39	6,0293	MFRF-M3	***1,62***	***72,20***	***7,6992***
	M4	FF-LSE -M4	3,00	67,16	6,0506	MFRF-M4	***1,68***	***66,18***	***8,5377***
	M5	FF-LSE -M5	***2,88***	***69,33***	***6,0355***	MFRF-M5	1,56	69,23	8,3100
	M6	FF-LSE -M6	***2,84***	***70,63***	***5,7873***	MFRF-M6	1,69	68,49	8,4176
6	M1	FF-LSE -M1	3,32	67,00	6,156	MFRF-M1	***1,56***	***72,30***	***7,7211***
	M2	FF-LSE -M2	3,22	68,35	6,0444	MFRF-M2	***1,48***	***73,75***	***7,4951***
	M3	FF-LSE -M3	3,22	68,57	6,026	MFRF-M3	***1,54***	***73,11***	***7,5549***
	M4	FF-LSE -M4	***3,26***	***66,98***	***6,134***	MFRF-M4	1,70	66,09	8,5481
	M5	FF-LSE -M5	***3,24***	***69,66***	***5,9346***	MFRF-M5	1,62	69,02	8,3400
	M6	FF-LSE -M6	***3,22***	***72,35***	***5,5205***	MFRF-M6	1,62	68,27	8,4497
7	M1	FF-LSE -M1	3,46	67,06	6,1184	MFRF-M1	***1,56***	***70,94***	***7,9098***
	M2	FF-LSE -M2	3,48	69,24	6,073	MFRF-M2	***1,48***	***72,43***	***7,6779***
	M3	FF-LSE -M3	3,48	69,22	6,0727	MFRF-M3	***1,56***	***72,23***	***7,6936***
	M4	FF-LSE -M4	***3,46***	***66,99***	***6,1264***	MFRF-M4	1,82	66,12	8,5429
	M5	FF-LSE -M5	***3,56***	***70,32***	***5,8796***	MFRF-M5	1,73	68,33	8,4400
	M6	FF-LSE -M6	***3,50***	***74,21***	***5,3946***	MFRF-M6	1,76	67,88	8,5063
8	M1	FF-LSE -M1	3,76	66,69	6,0946	MFRF-M1	***1,58***	***71,39***	***7,8385***
	M2	FF-LSE -M2	3,76	68,40	6,1141	MFRF-M2	***1,54***	***72,22***	***7,6953***
	M3	FF-LSE -M3	3,84	68,30	6,1767	MFRF-M3	***1,62***	***72,06***	***7,7055***
	M4	FF-LSE -M4	***3,74***	***66,61***	***6,0238***	MFRF-M4	1,90	66,14	8,5438
	M5	FF-LSE -M5	***3,76***	***70,66***	***5,8645***	MFRF-M5	1,71	68,25	8,4500
	M6	FF-LSE -M6	***3,58***	***71,50***	***5,8866***	MFRF-M6	1,64	67,77	8,5208
9	M1	FF-LSE -M1	4,10	66,94	6,1228	MFRF-M1	***1,60***	***71,53***	***7,8323***
	M2	FF-LSE -M2	4,14	68,69	6,0619	MFRF-M2	***1,48***	***73,09***	***7,5916***
	M3	FF-LSE -M3	4,08	68,65	6,1673	MFRF-M3	***1,58***	***72,78***	***7,6186***
	M4	FF-LSE -M4	***4,22***	***66,88***	***6,0948***	MFRF-M4	1,90	66,04	8,5522
	M5	FF-LSE -M5	***4,04***	***70,85***	***5,8765***	MFRF-M5	1,67	68,07	8,4700
	M6	FF-LSE -M6	***3,94***	***71,79***	***5,8028***	MFRF-M6	1,67	67,73	8,5261
10	M1	FF-LSE -M1	4,44	66,72	6,1073	MFRF-M1	***1,50***	***71,39***	***7,8579***
	M2	FF-LSE -M2	4,46	68,93	5,9515	MFRF-M2	***1,48***	***73,18***	***7,5880***
	M3	FF-LSE -M3	4,36	68,66	5,9582	MFRF-M3	***1,56***	***72,60***	***7,6402***
	M4	FF-LSE -M4	***4,40***	***66,72***	***6,0925***	MFRF-M4	1,94	66,09	8,5466
	M5	FF-LSE -M5	***4,46***	***70,61***	***5,8673***	MFRF-M5	1,82	68,02	8,4900
	M6	FF-LSE -M6	***4,06***	***71,46***	***5,7188***	MFRF-M6	1,93	67,74	8,5246

m, *RMSE* and R^2 represent average values obtained from 10 experiments; Bold numbers indicate optimal results for each cluster

Furthermore, optimal R^2 and *RMSE* values are calculated from the MFRF and FF-LSE for six models over 10 experiments with respect to 63 cases for different fuzzy clustering parameters c and m. These results are shown in Table 7 and Table 8 given in Appendix for training/testing datasets, respectively.

In this study, because of the non-normality of the R^2 and *RMSE* values, Mann-Whitney *U* test is used for comparison of the MFRF and FF-LSE. The null hypothesis (H_0) and the alternative hypothesis (H_1) which are formed to compare the FF-LSE and MFRF over 10 experiments considering each model, according to the R^2 and $RMSE$ are as follows:

$$H_0: \frac{1}{10}\sum_{cv=1}^{10} R^2_{j,cv} = \frac{1}{10}\sum_{cv=1}^{10} R^2_{k,cv}$$

$$H_1: \frac{1}{10}\sum_{cv=1}^{10} R^2_{j,cv} \neq \frac{1}{10}\sum_{cv=1}^{10} R^2_{k,cv}$$

and

$$H_0: \frac{1}{10}\sum_{cv=1}^{10} RMSE_{j,cv} = \frac{1}{10}\sum_{cv=1}^{10} RMSE_{k,cv}$$

$$H_1: \frac{1}{10}\sum_{cv=1}^{10} RMSE_{j,cv} \neq \frac{1}{10}\sum_{cv=1}^{10} RMSE_{k,cv}$$

The null hypothesis (H_0) indicates that the difference between the average values of optimal R^2 and *RMSE* obtained from the approach *j* and approach *k* over 10 experiments, is equal to 0. The alternative hypothesis (H_1) implies that the average of optimal R^2 and *RMSE* values of approach *j* and approach *k* over10 experiments are not equal. Accepting (**A**) the H_0 means that the two approaches are not significantly different while rejecting (**R**) the H_0 means that the approach *j* is significantly different from approach *k* with the 95% confidence level. Table 6 provides the Mann-Whitney *U* test results for comparison of these methods according to the considered six models for training / testing data sets, respectively.

Table 6. Mann Whitney *U* test results ($p<0.05$) for training / testing data sets.

Compared models		**Training data sets**		**Testing data sets**	
FF-LSE (*j*)	**MFRF (*k*)**	***R*²(%)**	***RMSE***	***R*²(%)**	***RMSE***
FF-LSE-M1	MFRF-M1	*p*=0.121 (**A**)	*p*=0.140 (**A**)	*p*=0.173 (**A**)	*p*=0.364 (**A**)
FF-LSE-M2	MFRF-M2	*p*=0.705 (**A**)	*p*=0.496 (**A**)	*p*=0.226 (**A**)	*p*=0.405 (**A**)
FF-LSE-M3	MFRF-M3	*p*=0.364 (**A**)	*p*=0.364 (**A**)	*p*=0.226 (**A**)	*p*=0.405 (**A**)
FF-LSE-M4	MFRF-M4	*p*=0.545 (**A**)	*p*=0.650 (**A**)	*p*=1.000 (**A**)	*p*=1.000 (**A**)
FF-LSE-M5	MFRF-M5	*p*=0.004 (**R**)	*p*=0.002 (**R**)	*p*=0.940 (**A**)	*p*=0.677 (**A**)
FF-LSE-M6	MFRF-M6	*p*=0.002 (**R**)	*p*=0.019 (**R**)	*p*=0.082 (**A**)	*p*=0.151 (**A**)

A indicates that null hypothesis (H_0) is accepted, **R** indicates that H_0 is rejected.

As can be seen from Table 6 for training data sets, the proposed MFRF approach is not significantly different from the FF-LSE approach for the models M1, M2, M3, and M4, since the null hypothesis is not rejected ($p>0.05$). However, the proposed MFRF approach is significantly different from the FF-LSE approach for the models M5 and M6, because the null hypothesis is rejected ($p<0.05$). The average of optimal R^2 and *RMSE* values show that the MFRF approach has better solutions than the FF-LSE for simple models (M1, M2, M3, M4) and has worse solutions for more complicated models (M5, M6) for training data sets. The reason is that more complicated models can lead to lower R^2 values and higher *RMSE* values due to computational difficulties for the MFRF.

According to the Mann Whitney *U* test results for testing data sets in Table 6, the proposed MFRF approach is not significantly different from the FF-LSE in terms of R^2 and *RMSE* values since the null hypothesis is accepted for all models (p>0.05). Therefore, the MFRF and FF-LSE could be used for both simple models and more complicated models for testing data sets. Detailed R^2 and *RMSE* values considering each model for training and testing data sets are shown in Table 7 and Table 8, respectively.

5. Conclusions

In this study, we proposed a new approach to Fuzzy System Models, namely the Multiplicative Fuzzy Regression Functions (MFRFs) using a new Multiplicative Fuzzy Clustering (MFC) algorithm. The novel fuzzy system modeling is structurally different from traditional fuzzy rule-based (FRB) approaches such as Zadeh [26]'s FRB, Takagi and Sugeno [27]'s FRB. It is also different from Tanaka *et al.* [28]'s fuzzy regression approach and Hathaway and Bezdek [5]'s fuzzy c regression method (FCRM), but it is similar to the Turksen [1]'s fuzzy functions with least squares errors (FF-LSE). The multiplicative fuzzy clustering (MFC) algorithm is an alternate representation of the IFC and FCM algorithm. In the MFRF approach, membership values obtained from the MFC algorithm and/or their transformations enter into a multiplicative input matrix together with original input variables for function identification with least squares errors technique.

In the application part, we applied the MFRF, FF-LSE and LSE approaches on Concrete Compressive Strength data set. To demonstrate the efficiency of the proposed MFRF approach, we compared the R^2 and *RMSE* values computed from the MFRF, FF-LSE, and LSE. According to these results, we showed that the proposed MFRF approach achieves better results than the FF-LSE approach for simple models, whereas yields worse results for more complicated models considering training data sets. However, there is no significant difference between the MFRF and FF-LSE for testing data sets. Both the MFRF and FF-LSE provide better results than LSE over all experiments for training/testing data sets.

In the future studies, we will focus on developing Cluster Validity Index for Multiplicative Fuzzy Clustering algorithm. Also, we will apply Multiplicative Fuzzy Regression Function on other real data sets for different complicated models.

Ethics statement All the authors also declare that there are no conflicts of interest to disclose.

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Appendix

A.1. Fuzzy C-Means Clustering

Fuzzy C-Means Clustering (FCM) clustering algorithm developed by Bezdek [4] partitions given data set into *c* clusters and tries to minimize an objective function as follows:

$$\begin{gathered} Min\, J_{FCM} = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m \left(\|x_k - v_i\| \right)_A \\ 0 \le \mu_{ik} \le 1,\ \forall i,k; \\ \sum_{i=1}^{c} \mu_{ik} = 1,\ \forall k; \\ 0 \le \sum_{k=1}^{n} \mu_{ik} \le n,\quad \forall i \end{gathered} \tag{17}$$

In Eq (17), J_{FCM} is an objective function to be minimized, $X = \{x_1, x_2, ..., x_n\}$ is the input matrix, x_k, $k = 1, 2, ..., n$ is the *k*th input vector and v_i, $i = 1, 2, ..., c$ is the *i*th cluster center. μ_{ik} denote the membership values of x_k belonging to v_i. *n* is total number of observations, *c* is total number of clusters, *m* is degree of fuzziness. $\|\cdot\|_A$indicates a norm that specifies a distance based similarity between the input vector x_k and the cluster centerv_i, $A = I$ is Euclidean norm and $A = C^{-1}$ is Mahalonobis norm.

The cluster centers and related membership values are calculated by solving constrained optimization problem in Eq (17) are given by Eq (18) and Eq (19), respectively:

$$v_{i,t} = \frac{\sum_{k=1}^{n} (u_{ik,t})^m x_k}{\sum_{k=1}^{n} (u_{ik,t})^m}. \tag{18}$$

$$\mu_{ik,t} = \left[\sum_{j=1}^{c} \left(\frac{\|x_k - v_{i,t-1}\|_A}{\|x_k - v_{j,t-1}\|_A} \right)^{2/m-1} \right]^{-1}. \tag{19}$$

For more details on the FCM algorithm, the readers can find it in [17, 25, 31].

A.2. Improved Fuzzy Clustering

The Improved Fuzzy Clustering (IFC) algorithm proposed by Celikyilmaz and Turksen [8] is a hybrid clustering method which combines Fuzzy c-Means Clustering (FCM) and Fuzzy c-Regression Models (FCRM), to find membership values as follows:

$$Min\ J_{IFC} = \sum_{i=1}^{c} \sum_{k=1}^{n} (\mu_{ik})^m \|x_k - v_i\|^2 + \sum_{i=1}^{c} \sum_{k=1}^{n} (\mu_{ik})^m \left(y_k - f_i \left(\tau_{ik}, \hat{\beta}_i \right)_{IFC} \right)^2. \tag{20}$$

where; $d_{ik} = \|x_k - v_i\|$ represents the distance from each x_k to each cluster center v_i. The first term in Eq (20) measures the clustering error and it is same as the FCM algorithm. The second term is squared error between y_k and interim fuzzy function-$f_i \left(\tau_{ik}, \hat{\beta}_i \right)_{IFC}$ of each cluster *i*, that is built using interim input matrix-τ_{ik}.

The initial membership values-μ_{ik} are calculated via FCM algorithm. The membership values μ_{ik} and/or their possible transformations are used as inputs to find the optimal membership values at each pair of (c, m).

An example of an interim input matrix could be given as follows:

$$\tau_i = \begin{bmatrix} 1 & \mu_{i1} & \mu_{i1}^2 & \cdots & e^{\mu_{i1}^m} \\ 1 & \mu_{i2} & \mu_{i2}^2 & \cdots & e^{\mu_{i2}^m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \mu_{in} & \mu_{in}^2 & \cdots & e^{\mu_{in}^m} \end{bmatrix}, k = 1, 2, ..., n.$$

The interim fuzzy function *i* which can be estimated by LSE is formed by using:

$$f_i\left(\tau_{ik}, \hat{\beta}_i\right)_{IFC} = \hat{\beta}_{i0} + \hat{\beta}_{i1}\mu_i + ... + \hat{\beta}_{i,nm}e^{\mu_i^m}\ ,\ i = 1, 2, ..., c. \tag{21}$$

In Eq (21), $\hat{\beta}_i = \left[\hat{\beta}_{i0}, \hat{\beta}_{i1}, ..., \hat{\beta}_{i,nm}\right]^T$ represents the regression parameter vector of the *i*th interim fuzzy function and *nm* is the total number of user-defined transformations of the membership values. Applying the Lagrangian multiplier method on Eq (20), improved membership values-μ_{ik}^{IFC} and improved cluster centers-v_i^{IFC} are calculated by Eq (22) and Eq (23), respectively:

$$\mu_{ik,t}^{IFC} = \left[\sum_{j=1}^{c}\left(\frac{\|x_k - v_{i,t-1}\|^2 + \left(y_k - f_i\left(\tau_{ik,t-1}, \hat{\beta}_i\right)\right)^2}{\|x_k - v_{j,t-1}\|^2 + \left(y_k - f_j\left(\tau_{jk,t-1}, \hat{\beta}_j\right)\right)^2}\right)^{1/m-1}\right]^{-1}. \tag{22}$$

$$v_{i,t}^{IFC} = \frac{\sum_{k=1}^{n}\left(\mu_{ik,t}^{IFC}\right)^m x_k}{\sum_{k=1}^{n}\left(\mu_{ik,t}^{IFC}\right)^m}. \tag{23}$$

The readers can find more details on IFC in [1, 8, 19, 24, 25].

Table 7. Optimal R^2(%), *RMSE* and (c*,m*) values obtained from the LSE, FF-LSE, and MFRF for each experiment of the training dataset .

	LSE			**FF-LSE**				**MFRF**			
Experiment	R^2	*RMSE*	**Model**	*c**	*m**	R^2	*RMSE*	*c**	*m**	R^2	*RMSE*
			M1	10,0	1,4	63,36	10,61	***10,0***	***1,4***	***68,11***	***9,90***
			M2	5,0	2,4	64,36	10,47	***10,0***	***1,4***	***67,95***	***9,93***
S1	61,8	10,90	M3	8,0	2,2	64,26	10,48	***10,0***	***1,4***	***68,00***	***9,92***
			M4	10,0	1,4	63,26	10,63	***2,0***	***1,4***	***63,51***	***10,60***
			M5	***10,0***	***2,4***	***89,44***	***5,69***	3,0	1,4	65,51	10,30
			M6	***5,0***	***1,4***	***65,01***	***10,37***	3,0	1,4	64,71	10,42
S2	64,6	10,50	M1	10,0	2,4	66,83	10,10	***9,0***	***1,4***	***68,88***	***9,79***
			M2	***6,0***	***2,6***	***72,23***	***9,24***	9,0	1,4	68,86	9,79
			M3	***3,0***	***1,4***	***70,45***	***9,53***	9,0	1,4	69,02	9,76
			M4	***2,0***	***2,6***	***66,98***	***10,08***	2,0	1,4	65,94	10,24
			M5	***3,0***	***1,4***	***70,55***	***9,52***	3,0	1,8	66,91	10,09
			M6	***10,0***	***1,8***	***70,72***	***9,49***	3,0	1,8	66,51	10,15
S3	69,3	9,80	M1	3,0	1,8	70,86	9,49	***10,0***	***1,4***	***72,65***	***9,20***
			M2	***3,0***	***1,4***	***73,89***	***8,98***	4,0	1,6	72,58	9,21
			M3	***3,0***	***1,4***	***73,31***	***9,08***	10,0	1,4	72,58	9,21
			M4	3,0	2,6	70,30	9,58	***2,0***	***1,4***	***71,01***	***9,47***
			M5	***3,0***	***1,4***	***74,10***	***8,95***	2,0	1,6	71,73	9,35
			M6	***6,0***	***2,4***	***88,48***	***5,97***	2,0	1,6	71,51	9,39
S4	67,3	9,59	M1	8,0	1,8	69,65	9,18	***3,0***	***1,6***	***71,38***	***8,92***
			M2	3,0	2,0	70,29	9,08	***3,0***	***1,6***	***72,30***	***8,78***
			M3	3,0	2,0	70,29	9,08	***3,0***	***1,6***	***72,14***	***8,80***
			M4	***8,0***	***1,8***	***69,59***	***9,19***	2,0	1,4	68,54	9,35
			M5	***7,0***	***2,6***	***80,11***	***7,43***	3,0	1,6	70,64	9,04
			M6	***7,0***	***2,6***	***80,11***	***7,43***	2,0	1,6	70,12	9,12
S5	60,30	9,51	M1	7,0	2,4	63,96	9,00	***3,0***	***1,4***	***66,55***	***8,68***
			M2	3,0	2,2	66,20	8,72	***3,0***	***1,4***	***66,60***	***8,67***
			M3	***3,0***	***2,2***	***66,76***	***8,64***	3,0	1,4	66,69	8,66
			M4	10,0	1,8	62,25	9,21	***3,0***	***1,4***	***62,89***	***9,14***
			M5	3,0	1,8	64,13	8,98	***3,0***	***1,4***	***65,89***	***8,76***
			M6	***9,0***	***2,6***	***81,14***	***6,51***	3,0	1,4	64,90	8,89
S6	57,70	9,86	M1	10,0	2,0	58,63	9,70	***3,0***	***1,4***	***64,13***	***9,04***
			M2	***9,0***	***2,0***	***66,70***	***8,70***	3,0	1,6	63,90	9,07
			M3	***9,0***	***2,0***	***66,70***	***8,70***	3,0	1,6	63,87	9,07
			M4	10,0	2,0	58,55	9,71	***3,0***	***1,4***	***60,58***	***9,47***
			M5	***5,0***	***2,4***	***80,19***	***6,71***	3,0	1,4	63,56	9,11
			M6	***7,0***	***2,4***	***86,73***	***5,49***	3,0	1,4	62,59	9,23
S7	64,60	10,50	M1	2,0	2,4	67,27	10,04	***6,0***	***1,4***	***68,49***	***9,85***
			M2	***8,0***	***2,0***	***69,07***	***9,76***	3,0	1,6	68,97	9,77
			M3	***6,0***	***2,4***	***81,24***	***7,60***	3,0	1,6	68,73	9,81
			M4	***5,0***	***1,4***	***66,41***	***10,17***	2,0	1,4	66, 15	10,21
			M5	***8,0***	***2,4***	***82,11***	***7,42***	3,0	1,4	67,70	9,97
			M6	***9,0***	***2,0***	***68,36***	***9,87***	3,0	1,4	67,12	10,06
S8	59,30	10,72	M1	***5,0***	***1,4***	***64,95***	***9,88***	7,0	1,4	63,36	10,11
			M2	***5,0***	***1,4***	***65,07***	***9,87***	7,0	1,4	63,36	10,11
			M3	***5,0***	***1,4***	***64,96***	***9,88***	7,0	1,4	63,47	10,09
			M4	***5,0***	***1,4***	***63,73***	***10,05***	2,0	1,6	60,21	10,53
			M5	***9,0***	***2,6***	***79,10***	***7,63***	3,0	1,6	61,26	10,39
			M6	***6,0***	***2,0***	***70,60***	***9,05***	3,0	1,6	61,00	10,43
S9	62,90	10,29	M1	4,0	1,4	64,58	9,99	***4,0***	***1,4***	**68,10**	***9,49***
			M2	***5,0***	***2,6***	***77,84***	***7,90***	4,0	1,4	68,05	9,49
			M3	***2,0***	***2,6***	***69,50***	***9,27***	4,0	1,4	68,33	9,45
			M4	***2,0***	***2,6***	***71,45***	***8,97***	2,0	1,4	64,77	9,97
			M5	***10,0***	***2,6***	***82,25***	***7,07***	2,0	1,4	66,31	9,75
			M6	***10,0***	***2,0***	***80,97***	***7,32***	2,0	1,4	65,80	9,82
S10	64,50	10,17	M1	2,0	1,6	66,10	9,89	***7,0***	***1,4***	***68,85***	***9,48***
			M2	3,0	1,8	67,89	9,62	***7,0***	***1,4***	***69,14***	***9,44***
			M3	***5,0***	***2,6***	***70,96***	***9,15***	7,0	1,4	69,24	9,42
			M4	3,0	1,4	65,25	10,01	***2,0***	***1,4***	***66,03***	***9,90***
			M5	***5,0***	***2,0***	***68,39***	***9,55***	2,0	1,4	66,61	9,82
			M6	***5,0***	***2,6***	***85,02***	***6,57***	2,0	1,4	66,53	9,83
Average	63,23	10,18	M1	6,10	1,86	65,62	9,79	6,20	1,42	**68,05**	**9,45**
			M2	5,00	2,04	***69,35***	**9,23**	5,30	1,48	68,17	9,43
			M3	4,70	2,02	***69,84***	***9,14***	5,90	1,46	68,21	9,42
			M4	5,80	1,90	***65,78***	***9,76***	2,20	1,42	64,83	9,89
			M5	6,30	2,16	***77,04***	***7,90***	2,70	1,50	66,61	9,66
			M6	7,40	2,18	***77,71***	***7,81***	2,60	1,50	66,08	9,73

Table 8. Optimal $R^2(\%)$, *RMSE* and (c^*,m^*) values obtained from the LSE, FF-LSE, and MFRF for each experiment of the testing dataset .

	LSE			FF-LSE				MFRF			
Experiment	R^2	*RMSE*	**Model**	c^*	m^*	R^2	*RMSE*	c^*	m^*	R^2	*RMSE*
			M1	6,0	1,4	85,00	4,78	***4,0***	***1,6***	***85,39***	***4,72***
			M2	***6,0***	***1,4***	***85,91***	***4,64***	4,0	1,6	85,49	4,71
S1	84,20	5,01	M3	***6,0***	***1,4***	***85,93***	***4,63***	4,0	1,6	85,49	4,71
			M4	***5,0***	***2,6***	***85,07***	***4,77***	4,0	1,4	84,69	4,83
			M5	***7,0***	***2,0***	***88,84***	***4,13***	4,0	1,4	85,06	4,78
			M6	***6,0***	***2,4***	***92,02***	***3,49***	4,0	1,4	84,93	4,80
S2	71,70	6,87	M1	4,0	2,0	72,64	6,61	***4,0***	***1,4***	***79,67***	***5,70***
			M2	4,0	1,8	73,82	6,46	***4,0***	***1,4***	***79,86***	***5,67***
			M3	4,0	1,8	73,86	6,46	***4,0***	***1,4***	***79,77***	***5,68***
			M4	4,0	2,0	72,61	6,61	***2,0***	***1,4***	***72,90***	***6,58***
			M5	2,0	2,6	75,77	6,22	***3,0***	***1,4***	***76,14***	***6,17***
			M6	***7,0***	***2,4***	***86,25***	***4,68***	2,0	1,4	74,62	6,36
S3	52,90	8,79	M1	9,0	2,0	54,11	8,49	***3,0***	***1,4***	***59,83***	***7,94***
			M2	9,0	2,0	54,52	8,45	***3,0***	***1,4***	***61,59***	***7,76***
			M3	9,0	2,0	54,52	8,45	***3,0***	***1,4***	***60,94***	***7,83***
			M4	***9,0***	***2,0***	***53,95***	***8,50***	6,0	2,0	53,36	8,55
			M5	***10,0***	***2,6***	***58,64***	***8,06***	3,0	1,4	55,76	8,33
			M6	***7,0***	***2,2***	***77,77***	***5,91***	3,0	1,4	53,88	8,51
S4	45,10	11,12	M1	9,0	2,0	54,11	8,49	***3,0***	***1,4***	***59,83***	***7,94***
			M2	9,0	2,0	54,52	8,45	***3,0***	***1,4***	***61,59***	***7,76***
			M3	9,0	2,0	54,52	8,45	***3,0***	***1,4***	***60,94***	***7,83***
			M4	***9,0***	***2,0***	***53,95***	***8,50***	6,0	2,0	53,36	8,55
			M5	***10,0***	***2,6***	***58,64***	***8,06***	3,0	1,4	55,76	8,33
			M6	***7,0***	***2,2***	***77,77***	***5,91***	3,0	1,4	53,88	8,51
S5	49,30	12,67	M1	5,0	2,2	53,59	11,86	***2,0***	***1,4***	***71,44***	***9,30***
			M2	7,0	2,6	59,33	11,10	***2,0***	***1,4***	***71,54***	***9,29***
			M3	7,0	2,6	59,58	11,07	***3,0***	***1,6***	***71,27***	***9,33***
			M4	5,0	2,2	53,87	11,82	***2,0***	***1,4***	***59,85***	***11,03***
			M5	9,0	2,6	60,55	10,93	***2,0***	***1,4***	***68,27***	***9,81***
			M6	10,0	1,8	60,65	10,92	***2,0***	***1,4***	***65,79***	***10,18***
S6	68,00	11,32	M1	6,0	2,4	69,62	10,78	***2,0***	***1,4***	***78,63***	***9,04***
			M2	10,0	1,4	73,94	9,98	***2,0***	***1,4***	***79,31***	***8,90***
			M3	10,0	1,4	73,90	9,99	***2,0***	***1,4***	***79,15***	***8,93***
			M4	2,0	2,6	69,42	10,82	***2,0***	***1,4***	***72,03***	***10,34***
			M5	3,0	2,2	74,05	9,96	***2,0***	***1,4***	***77,02***	***9,38***
			M6	3,0	1,4	74,77	9,82	***2,0***	***1,4***	***75,57***	***9,67***
S7	73,70	6,59	M1	4,0	1,6	75,71	6,20	***3,0***	***2,6***	***80,92***	***5,49***
			M2	4,0	1,6	75,50	6,23	***4,0***	***1,4***	***81,05***	***5,48***
			M3	4,0	1,6	75,50	6,23	***4,0***	***1,4***	***81,00***	***5,48***
			M4	***4,0***	***1,6***	***75,16***	***6,27***	3,0	1,8	74,30	6,38
			M5	4,0	2,4	77,15	6,01	***3,0***	***1,4***	***77,18***	***6,01***
			M6	***8,0***	***2,4***	***78,48***	***5,84***	3,0	1,4	75,47	6,23
S8	69,00	8,86	M1	3,0	1,4	70,80	8,42	***6,0***	***1,4***	***75,13***	***7,77***
			M2	5,0	1,4	72,75	8,13	***2,0***	***1,6***	***75,51***	***7,71***
			M3	5,0	1,4	72,79	8,12	***9,0***	***1,4***	***75,49***	***7,71***
			M4	3,0	1,4	69,88	8,55	***3,0***	***1,6***	***71,75***	***8,28***
			M5	9,0	1,4	72,60	8,15	***3,0***	***1,6***	***74,25***	***7,90***
			M6	***10,0***	***1,4***	***74,78***	***7,82***	3,0	1,6	73,55	8,01
S9	64,20	9,47	M1	10,0	1,6	67,71	8,80	***4,0***	***1,4***	***74,44***	***7,83***
			M2	7,0	2,0	69,99	8,49	***4,0***	***1,4***	***75,18***	***7,72***
			M3	7,0	2,0	69,99	8,49	***4,0***	***1,4***	***75,07***	***7,74***
			M4	2,0	1,4	67,80	8,79	***2,0***	***1,6***	***68,92***	***8,64***
			M5	***9,0***	***2,2***	***75,39***	***7,69***	2,0	1,6	71,70	8,24
			M6	***9,0***	***2,2***	***75,33***	***7,70***	2,0	1,6	70,93	8,35
S10	78,10	7,28	M1	***9,0***	***2,2***	***79,09***	***6,96***	2,0	1,6	71,62	8,25
			M2	***4,0***	***2,6***	***81,49***	***6,55***	4,0	1,4	79,52	6,88
			M3	***3,0***	***1,6***	***80,86***	***6,66***	4,0	1,4	79,55	6,88
			M4	***3,0***	***2,2***	***79,20***	***6,94***	2,0	1,4	78,79	7,01
			M5	***10,0***	***2,4***	***85,66***	***5,76***	3,0	1,6	78,75	7,01
			M6	***2,0***	***2,4***	***84,16***	***6,05***	2,0	1,4	78,85	7,00
Average	65,62	8,80	M1	6,50	1,88	68,24	8,14	3,30	1,56	***73,69***	***7,40***
			M2	6,50	1,88	70,18	7,85	3,20	1,44	***75,06***	***7,19***
			M3	6,40	1,78	70,15	7,86	4,00	1,44	***74,87***	***7,21***
			M4	4,60	2,00	68,09	8,16	3,20	1,60	***69,00***	***8,02***
			M5	7,30	2,30	***72,73***	***7,50***	2,80	1,46	71,99	7,60
			M6	6,90	2,08	***78,20***	***6,81***	2,60	1,44	70,75	7,76