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Hybrid Data-Driven Active Disturbance Rejection Sliding Mode Control with Tower Crane Systems Validation

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Abstract. This paper proposes a combination of a data-driven algorithm represented by continuous-time *Active Disturbance Rejection Control* (ADRC) and a *Sliding Mode Control* (SMC) algorithm. The purpose of this hybrid controller referred to as ADRC-SMC is to improve the overall control-loop system performance while guaranteeing its stability. This will be done through clear, simple, and transparent steps of controller design in a novel real formulation focused on practical implementation. The parameters of the novel second-order continuous-time ADRC-SMC algorithm are optimally tuned using a metaheuristic slime mould algorithm. The purpose of obtaining the parameters of the ADRC-SMC algorithms in this model-based manner is to reduce the heuristics and further ensure a fair performance comparison of the ADRC-SMC algorithm with that of the popular ADRC algorithm. The data-driven second-order continuous-time ADRC and ADRC-SMC algorithms are validated experimentally validated on tower crane laboratory equipment.

Key-words: Active disturbance rejection control system structure; sliding mode control; slime mould algorithm; tower crane system; real-time experiments.

1. Introduction

Data-driven control algorithms [1] got a head start in the past two decades to solve complex control problems when the mathematical model of the process is unknown, or its nonlinearities are very hard to catch through a nonlinear mathematical model. Currently, data-driven control algorithms are a topic of interest in both academia and industry. They passed the theoretical stage, and they have been currently validated in practice on different types of processes. The main feature of data-driven control algorithms is that the parameter tuning is carried out using the input/output data of the process without needing other information about the plant. According to the authors' opinion, some of the most successful data-driven control algorithms are *active disturbance rejection control* (ADRC) [2], model-free adaptive control [3], model-free control [4], virtual reference feedback tuning [5], iterative feedback tuning [6], data-driven model-reference control [7], and model-free optimal control [8].

The ADRC algorithm is built around a *proportional-derivative* (PD) controller and uses an extended state observer to estimate the process outputs and mainly the unknown disturbance term, and it does not depend on an accurate process model. The total disturbance is compensated by state error feedback control law via the extended state observer. Unfortunately, the ADRC algorithm as it was initially proposed has two main weak points, (1) and (2): (1) The gains K_1 and K_2 as they will further be referred to in the following sections have no consistent guidelines in computing and determining them. In the current state-of-the-art, most authors prefer to heuristically choose these two parameters (namely gains), or they choose to determine the parameters optimally. The heuristics of parameter choosing is advantageous if the controlled process is unknown, but it comes with the disadvantage that it is very hard for the practitioner to directly find the parameters that ensure the best control system performance. (2) The stability analysis of the closed-loop control system is complicated since it is not based on accurate mathematical models of the process.

The stability analysis will be performed in the current paper in terms of considering the closed-loop error dynamics and the dynamic model of the observer. The dynamics of the filters that produce the estimated derivatives of the reference inputs are not included.

After its initial formulation, the ADRC algorithm was continuously improved and validated on numerous processes with stability guarantee including rotary DC motor by linear ADRC using anti-windup strategy and Hurwitz polynomials [12], ball and beam systems control using Routh's stability criterion [13], noncircular turning processes using vector margin variation rate [14], electronic gearboxes with ADRC parameters optimally tuned via PSO algorithm, control of all axes of CNC machines [15], *tower crane systems* (TCS) control by fuzzy ADRC with parameters tuned via virtual reference feedback tuning [16], TCS using virtual reference feedback tuning combined with ADRC [17], twin rotor aerodynamic systems by ADRC mixed with virtual reference feedback tuning [18], data-driven control containing an estimator similar to ADRC applied to shape memory alloys processes [19], also subjected to ADRC [20].

The well-known *sliding mode control* (SMC) algorithms are nonlinear and relatively simple, with the main advantage of robustness against parameter variations and disturbances. During the past decades, data-driven control algorithms have been combined with SMC to guarantee stability and improve the overall control system performance. Among the most representative recent data-driven SMC combinations are the mix of model-free control and SMC with experimental validation are the mix of model-free control and SMC with experimental validation on twin rotor aerodynamic system [21], the mix of data-driven adaptive control and SMC to control shape memory alloy actuators [22], the mix of H_∞ and SMC controlling persistent dwell-time systems under the dynamic event-triggered mechanism [23]. In this regard, various combinations of ADRC algorithms and SMC algorithms have been proposed to improve the overall ADRC performance and guarantee the closed-loop control system stability using Lyapunov's theory. The combinations of ADRC and SMC are validated on different processes including optoelectronic platforms [24], servo systems [25], and quadrotor systems [26].

This paper proposes to overcome the ADRC's two main weak points, namely the heuristic tuning of the gains and the closed-loop control system stability guarantee, by mixing the second-order continuous-time ADRC algorithm and an SMC algorithm. This novel combination of ADRC and SMC, referred to as the second-order continuous-time ADRC-SMC algorithm, has the advantages (i) to (iv) versus the state-of-the-art briefly discussed above: (i) It uses the reaching and existence conditions of SMC to guarantee the control system stability. (ii) It offers clear, simple, and transparent steps to implement the controllers in terms of novel implemented control laws, which are different from the theoretical ones by including the dynamics of the filters that produce the estimated derivatives of the reference inputs. (iii) The parameters of the continuous-time ADRC-SMC algorithm are optimally tuned using metaheuristic SMA. This approach reduces the heuristics and also ensures a fair comparison of the performance of the ADRC-SMC algorithm with that of the ADRC algorithm. (iv) Although the weak point (2) is not fully mitigated, a step forward is achieved expressed as stability analyses without considering the filter dynamics. The stability conditions are expressed as feasible domains in the optimization problems.

Both algorithms discussed in this paper are validated through experiments on the tower crane laboratory equipment. The performance comparison is quantitative and shows the performance enhancement ensured by the ADRC-SMC algorithm proposed here.

The rest of the paper is structured as follows. The continuous-time ADRC algorithm and the novel continuous-time ADRC-SMC algorithm are described in Section 2. The tower crane system laboratory equipment is presented in Section 3. The experimental setup where ADRC's and ADRC-SMC's optimal parameters are tuned using metaheuristic SMA, and the results are shown in Section 4. The conclusions are highlighted in Section 5.

2. The Second-order Continuous-time ADRC and ADRC-SMC Algorithms

2.1. Second-order continuous-time ADRC algorithm

The design of the second-order ADRC algorithm detailed in [27] is based on the disturbed double integrator process model, obtained after substituting $f(t)$ from (s3) in (s1) given in [27]:

$$\ddot{y}(t) = b_0 u(t) + f(t). \tag{1}$$

The theoretical continuous-time control law specific to ADRC, which is in fact a modified PD controller, is [12, 14, 16]

$$\begin{aligned} u(t) &= [K_1(r(t) - \hat{z}_1(t)) + K_2(\dot{r}(t) - \hat{z}_2(t)) + \ddot{r}(t) - \hat{z}_3(t)]/b_0 \\ &= [K_1\hat{e}(t) + K_2\hat{\dot{e}}(t) + \ddot{r}(t) - \hat{z}_3(t)]/b_0, \end{aligned} \tag{2}$$

where t is the continuous time index, $r(t)$ is the reference input (or the set-point), namely the controlled output trajectory imposed to the control system, $\hat{e}(t) = r(t) - \hat{y}(t) = r(t) - \hat{z}_1(t)$ is the estimated control error, $e(t) = r(t) - y(t)$ is the control error, $\hat{e}(t) = \dot{r}(t) - \hat{\dot{y}}(t) = \dot{r}(t) - \hat{z}_2(t)$ is the estimate of the first-order time derivative of the control error $\dot{e}(t) = \dot{r}(t) - \dot{y}(t)$, and K_1 and K_2 are the gains of the PD component of the ADRC control law, namely $[K_1\hat{e}(t) + K_2\hat{\dot{e}}(t)]/b_0$.

The first-order derivative plus low-pass filter with the following transfer function is proposed in [21] to implement the computation of $\dot{r}(t)$ using $r(t)$:

$$H_{DLP}(s) = s/(1 + T_{DLP}s), \tag{3}$$

where $T_{DLP} > 0$ is the filter time constant. The input of the filter is $r(t)$ and the output of the filter is the estimated $\dot{r}(t)$. The filter with the transfer function in (3), which is actually a real derivative element, is also used, with the input $\dot{r}(t)$ and the output $\ddot{r}(t)$, to implement the computation of $\ddot{r}(t)$, which is employed in (2). However, in order to make clear differences of the notations used above in theory, and the notations used in the practical implementation, the estimates of $\dot{r}(t)$ and $\ddot{r}(t)$ generated using the filter in (3) will be denoted as $\tilde{\dot{r}}(t)$ and $\tilde{\ddot{r}}(t)$, respectively. Therefore, the implemented control law will be expressed as follows after rewriting the theoretical control law in (2) and using the above notations:

$$\begin{aligned} u(t) &= [K_1(r(t) - \hat{z}_1(t)) + K_2(\tilde{\dot{r}}(t) - \hat{z}_2(t)) + \tilde{\ddot{r}}(t) - \hat{z}_3(t)]/b_0 \\ &= [K_1\hat{e}(t) + K_2\hat{\dot{e}}(t) + \tilde{\ddot{r}}(t) - \hat{z}_3(t)]/b_0, \end{aligned} \tag{4}$$

where the estimate of the first-order time derivative of the control error used in the implemented control law is $\hat{\dot{e}}(t) = \tilde{\dot{r}}(t) - \hat{\dot{y}}(t) = \tilde{\dot{r}}(t) - \hat{z}_2(t)$.

Using (4), the block diagram of the control system with the second-order continuous-time ADRC algorithm is presented in Fig. 1 given in [27].

The gains K_1 and K_2 of the PD component of the ADRC control law in (4) are usually determined according to [12, 14, 16]. However, to reduce the heuristics in the tuning carried out using the above mentioned approaches and caused by choosing the values of continuous-time ADRC algorithm parameters K_1 and K_2, included in the parameter vector $\mathbf{K}_{\text{ADRC}} = [K_1\ K_2]^T$ this paper suggests the calculation of the optimal parameter vector $\mathbf{K}^*_{\text{ADRC}} = [K_1^*\ K_2^*]^T$ as the solution to the optimization problem

$$\mathbf{K}^*_{\text{ADRC}} = \underset{\mathbf{K}_{\text{ADRC}} \in D_{\mathbf{K}_{\text{ADRC}}}}{\arg\min}\ J_{\mathbf{K}_{\text{ADRC}}}(\mathbf{K}_{\text{ADRC}}),\ J_{\mathbf{K}_{\text{ADRC}}}(\mathbf{K}_{\text{ADRC}}) = \int_{t_0}^{t_f} (e(t, \mathbf{K}_{\text{ADRC}}))^2 dt, \tag{5}$$

where $\mathbf{K}_{\text{ADRC}}$ is the vector variable of the cost function $J_{\mathbf{K}_{\text{ADRC}}}$, $[t_0, t_f]$ is the time horizon, and $D_{\mathbf{K}_{\text{ADRC}}}$ is the feasible domain of $\mathbf{K}_{\text{ADRC}}$.

The statement and the proof of Theorem 1 are given in [27].

The dynamic regime involved in the closed-loop experiment that allows the evaluation of the cost function $J_{\mathbf{K}_{\text{ADRC}}}$ in the optimization problem defined in (5) is the nominal operating regime of the control system. More details regarding the dynamic regime are given in [27].

The design strategy of the second-order continuous-time ADRC algorithm is summarized in the following steps:

Step ADRC-1. Specify the dynamic regime involved in the open-loop experiment and solve the optimization problem defined in (s5) given in [27] accounting for (s6) given in [27] to compute the optimal *extended Luenberger state observer* (ELSO) gain matrix $\mathbf{L}^* = [l_1^*\ \ l_2^*\ \ l_3^*\ \ ]^T$.

Step ADRC-2. Choose the value $b_0 \neq 0$ of the estimate of the coefficient of the control input.

Step ADRC-3. Set the value of the parameter $T_{DLP} > 0$ of the first-order derivative plus low-pass filter with the transfer function in (3) to implement the computation of $\dot{\tilde{r}}$ and $\ddot{\tilde{r}}$ in the control law illustrated in Fig. 1 given in [27]. This parameter value is set as a compromise to noise reduction and the delay induced by the filter.

Step ADRC-4. Specify the dynamic regime involved in solving the optimization problem in (5) and the evaluation of the cost function $J_{\mathbf{K}_{\text{ADRC}}}$, and solve the optimization defined in (5) accounting for (s7) given in [27] to compute the optimal parameter vector $\mathbf{K}^*_{\text{ADRC}} = [K_1^* \quad K_2^*]^T$.

2.2. Second-order continuous-time ADRC-SMC approach

Starting with the continuous-time ADRC control law in (2) and adding the augmented control input $u_{aug}(t)$, the continuous-time second-order ADRC-SMC control law is expressed as

$$u(t) = [K_1\hat{e}(t) + K_2\hat{\dot{e}}(t) + \ddot{r}(t) - \hat{z}_3(t)]/b_0 + u_{aug}(t). \tag{6}$$

To derive the dynamics equation of the closed-loop control system, $u(t)$ is substituted from (6) in the disturbed double integrator process model in (1) leading to

$$\ddot{y}(t) = K_1\hat{e}(t) + K_2\hat{\dot{e}}(t) + \ddot{r}(t) + b_0 u_{aug}(t) + f(t) - \hat{z}_3(t). \tag{7}$$

The estimation error of the unknown disturbance term $f(t)$, with the notation $e_{est}(t)$, is defined as

$$e_{est}(t) = f(t) - \hat{f}(t) = f(t) - \hat{z}_3(t), \tag{8}$$

where $\hat{f}(t) = \hat{z}_3(t)$ is the estimated disturbance term. Taking into consideration the expression of the time derivative of the control error $\ddot{e}(t) = \ddot{r}(t) - \ddot{y}(t)$ and substituting (8) modified as $f(t) - \hat{z}_3(t) = e_{est}(t)$ in (7), the dynamics of the closed-loop control system becomes

$$\ddot{e}(t) + K_2\hat{\dot{e}}(t) + K_1\hat{e}(t) + b_0 u_{aug}(t) + e_{est}(t) = 0. \tag{9}$$

For merging the continuous-time ADRC algorithm with sliding mode control, the following state variables are introduced:

$$x_1(t) = \hat{e}(t), x_2(t) = \hat{\dot{e}}(t), x_3(t) = e(t), x_4(t) = \dot{e}(t), \tag{10}$$

resulting that

$$\dot{x}_1(t) = x_2(t), \dot{x}_3(t) = x_4(t). \tag{11}$$

Substituting the control errors, the estimated control errors, and their first-order time derivatives expressed in (10) and (11) in the closed-loop control system dynamics in (9) leads to

$$\dot{x}_4(t) = -K_1 x_1(t) - K_2 x_2(t) - b_0 u_{aug}(t) - e_{est}(t). \tag{12}$$

Rewriting the disturbed double integrator process model in (1) using estimates

$$\hat{\ddot{y}}(t) = b_0 u(t) + \hat{f}(t), \tag{13}$$

subtracting (1) from (13), and next adding $\ddot{r}(t)$ to the right-hand and left-hand terms, it turns out that

$$\hat{\ddot{e}}(t) = \ddot{e}(t) + e_{est}(t). \tag{14}$$

The derivatives in (10) are next substituted in (14) leading to

$$\dot{x}_2(t) = \dot{x}_4(t) + e_{est}(t), \tag{15}$$

and finally $\dot{x}_4(t)$ is substituted from (12) in (15) resulting in

$$\dot{x}_2(t) = -K_1x_1(t) - K_2x_2(t) - b_0u_{aug}(t). \tag{16}$$

Thus, using (11), (12), and (16), the state-space equations of the closed-loop control system dynamics in (9) become

$$\begin{aligned} &\dot{x}_1(t) = x_2(t), \dot{x}_2(t) = -K_1x_1(t) - K_2x_2(t) - b_0u_{aug}(t), \\ &\dot{x}_3(t) = x_4(t), \dot{x}_4(t) = -K_1x_1(t) - K_2x_2(t) - b_0u_{aug}(t) - e_{est}(t). \end{aligned} \tag{17}$$

The switching variable of the sliding mode control law $u_{aug}(t)$ is defined as $\sigma(t)$

$$\sigma(t) = x_1(t) + T_1x_2(t) + \beta x_3(t) + \beta T_2x_4(t), \tag{18}$$

where T_1, T_2 and β are three positive design parameters that establish the proper behavior of the control system on the sliding hyper-plane with the equation $\sigma(t) = 0$. The stability of the closed-loop control system dynamics in (17) is guaranteed by first introducing the following Lyapunov function candidate:

$$V(t) = 0.5\sigma^2(t). \tag{19}$$

Next, in the context of Lyapunov's second method of stability, the condition $\dot{V}(t) < 0$ is updated into the sliding mode reaching and existence condition

$$\dot{V}(t) = \sigma(t)\dot{\sigma}(t) < 0, \ \sigma(t) \neq 0, \tag{20}$$

which is used in the derivation of the control input $u_{aug}(t)$ as function of $x_1(t), x_2(t), x_3(t)$ and $x_4(t)$.

The first-order time derivative of the switching variable, $\dot{\sigma}(t)$, is obtained from (18):

$$\dot{\sigma}(t) = \dot{x}_1(t) + T_1\dot{x}_2(t) + \beta\dot{x}_3(t) + \beta T_2\dot{x}_4(t). \tag{21}$$

Substituting the derivatives of the state variables expressed in (17) in (21), $\dot{\sigma}(t)$ becomes

$$\begin{aligned} \dot{\sigma}(t) = &-K_1(T_1 + \beta T_2)x_1(t) + (1 - T_1K_2 - \beta T_2K_2)x_2(t) + \beta x_4(t) \\ &- b_0(T_1 + \beta T_2)u_{aug}(t) - \beta T_2e_{est}(t). \end{aligned} \tag{22}$$

The estimation error of the unknown disturbance term $e_{est}(t)$ is unknown, and it is assumed to be bounded in terms of

$$|e_{est}(t)| \le e_{est\,\max}, \tag{23}$$

where $e_{est\,\max} = \text{const} > 0$ is a design parameter estimated by the control systems designer, and it represents the upper bound of $|e_{est}(t)|$.

A continuous-time ADRC-SMC approach is proposed and described as follows. The ADRC-SMC acronym will be further used for the continuous-time ADRC-SMC approach. Within this approach, the augmented control input is composed of two control inputs

$$u_{aug}(t) = u_{eq}(t) + u_{cor}(t), \tag{24}$$

where $u_{eq}(t)$ is the equivalent control input and $u_{cor}(t)$ is the correction control input. The first control input, $u_{eq}(t)$, is computed using the ideal sliding mode condition, namely $\sigma(t) = 0$, which leads to

$$\dot{\sigma}(t) = 0. \tag{25}$$

Substituting $u_{aug}(t) = u_{eq}(t)$ in (22) and solving (25) for $u_{eq}(t)$, the expression of $u_{eq}(t)$ is

$$\begin{aligned} u_{eq}(t) = {} & [-K_1(T_1 + \beta T_2)x_1(t) + (1 - T_1K_2 - \beta T_2K_2)x_2(t) + \beta x_4(t) - \beta T_2 e_{est}(t)]/ \\ & [b_0(T_1 + \beta T_2)]. \end{aligned} \tag{26}$$

Since $e_{est}(t)$ in (26) is unknown, $e_{est}(t)$ will be replaced with $e_{est\,\max}$, and (26) will become

$$\begin{aligned} u_{eq}(t) = {} & [-K_1(T_1 + \beta T_2)x_1(t) + (1 - T_1K_2 - \beta T_2K_2)x_2(t) + \beta x_4(t) - \beta T_2 e_{est\,\max}]/ \\ & [b_0(T_1 + \beta T_2)]. \end{aligned} \tag{27}$$

The replacement of (26) with (27) is justified by the presence of $u_{cor}(t)$ as it is well-accepted that SMC has certain robustness properties. Also, SMC can tackle mismatched disturbances.

A boundary layer approach is next applied to fulfill the sliding mode reaching and existence condition in (20) and for alleviating the chattering effects. Thus, the following expression of $u_{cor}(t)$ is proposed:

$$u_{cor}(t) = \eta/[b_0(T_1 + \beta T_2)]\text{sat}(\sigma(t), \varepsilon) = \eta/[b_0(T_1 + \beta T_2)] \begin{cases} -1 & \text{if} \sigma(t) < -\varepsilon, \\ \sigma(t)/\varepsilon & \text{if} |\sigma(t)| \le \varepsilon, \\ 1 & \text{if} \sigma(t) > \varepsilon, \end{cases} \tag{28}$$

where $\eta > 0$ is the convergence factor and $\varepsilon > 0$ is the boundary layer thickness.

The expression of $\dot{V}(t)$ is computed after performing the substitution of $u_{eq}(t)$ in (27) and $u_{cor}(t)$ in (28) in (24), and afterwards in (22) using (20):

$$\dot{V}(t) = \sigma(t)\dot{\sigma}(t) = -\sigma(t)\eta\,\text{sat}(\sigma(t), \varepsilon) + \sigma(t)\beta T_2(e_{est\,\max} - e_{est}(t)). \tag{29}$$

The statement and the proof of Theorem 2 is given in [27].

The expression of the switching variable dynamics in sliding mode is obtained as follows from (29) in terms of dropping out the multiplying term $\sigma(t)$:

$$\dot{\sigma}(t) = -\eta\,\mathrm{sat}(\sigma(t),\varepsilon) + \beta T_2(e_{est\,\max} - e_{est}(t)). \tag{30}$$

The value of the time derivative of the steady-state switching variable σ_∞, as a particular expression of (25), is $\dot{\sigma}_\infty = 0$ which corresponds to a relatively small constant value of σ_∞, meaning that case 1 is employed in steady-state regimes, namely $|\sigma_\infty| \leq \varepsilon$, with the ideal value $\sigma_\infty = 0$, therefore $\mathrm{sat}(\sigma_\infty,\varepsilon) = \sigma_\infty/\varepsilon$ via (28), the steady-state counterpart of the switching variable dynamics in (30) becomes

$$0 = -\eta\sigma_\infty/\varepsilon + \beta T_2(e_{est\,\max} - e_{est\infty}), \tag{31}$$

where $e_{est\infty}$ is the steady-state estimation error and $e_{est\,\max}$. Thus (31) returns the following steady-state switching variable:

$$\sigma_\infty = (\varepsilon\beta T_2/\eta)(e_{est\,\max} - e_{est\infty}). \tag{32}$$

Similar to (s51) given in [27], a useful recommendation for practitioners resulting from (32) is to use accurate derivative estimators, large values of $\eta > 0$, small values of $\varepsilon > 0$ and small values of $\beta > 0$, for reducing the steady-state effects of the system dynamics in sliding mode.

Substituting the equivalent control input $u_{eq}(t)$ from (27) and the correction control input $u_{cor}(t)$ from (28) in (24), the augmented control input $u_{aug}(t)$ becomes

$$\begin{aligned} u_{aug}(t) = [&-K_1(T_1+\beta T_2)x_1(t) + (1 - T_1K_2 - \beta T_2K_2)x_2(t) + \beta x_4(t) - \beta T_2 e_{est\,\max} \\ &+ \eta\,\mathrm{sat}(\sigma(t),\varepsilon)]/[b_0(T_1+\beta T_2)]. \end{aligned} \tag{33}$$

Substituting $u_{aug}(t)$ expressed in (33) in (6) and using the notations of the state variables in (10), the resulting expression of the theoretical control law of the ADRC-SMC structure is

$$u(t) = (\ddot{r}(t) - \hat{z}_3(t))/b_0 + [\hat{\dot{e}}(t) + \beta\dot{e}(t) - \beta T_2 e_{est\,\max} + \eta\,\mathrm{sat}(\sigma(t),\varepsilon)]/[b_0(T_1+\beta T_2)]. \tag{34}$$

Summarizing the relations presented in this sub-section, the block diagram of the control system with the continuous-time ADRC-SMC algorithm is presented in Fig. 2 given in [27], which employs the following expression of $\sigma(t)$, resulted after substituting in (18) the state variables defined in (10):

$$\sigma(t) = \hat{e}(t) + T_1\hat{\dot{e}}(t) + \beta e(t) + \beta T_2\dot{e}(t). \tag{35}$$

Proceeding as in the case of the continuous-time control law specific to ADRC, to make clear differences of the notations used above in theory, and the notations used in the practical implementation, the estimates of $\dot{r}(t)$, $\ddot{r}(t)$, $\dot{y}(t)$ and $\dot{e}(t)$ generated using the first-order derivative plus low-pass filter in (3) will be denoted as $\tilde{\dot{r}}(t)$, $\tilde{\ddot{r}}(t)$, $\tilde{\dot{y}}(t)$ and $\tilde{\dot{e}}(t)$, respectively. Therefore, the implemented control law will be expressed as follows after rewriting the theoretical control law in (34) and using the above notations:

$$u(t) = (\tilde{\ddot{r}}(t) - \hat{z}_3(t))/b_0 + [\hat{\dot{e}}(t) + \beta\tilde{\dot{e}}(t) - \beta T_2 e_{est\,\max} + \eta\,\mathrm{sat}(\sigma(t),\varepsilon)]/[b_0(T_1+\beta T_2)]. \tag{36}$$

Similarly, the implemented expression of $\sigma(t)$ will be expressed as follows after rewriting the switching variable $\sigma(t)$ according to (35) and using the above notations based on the outputs of the first-order derivative plus a low-pass filter in (3):

$$\sigma(t) = \hat{e}(t) + T_1\hat{\dot{e}}(t) + \beta e(t) + \beta T_2 \tilde{\dot{e}}(t). \tag{37}$$

Using (36) and (37), the block diagram of the control system with the second-order continuous-time ADRC-SMC algorithm is presented in Fig. 2 given in [27].

However, to reduce the heuristics in the design caused by choosing the values of continuous-time ADRC-SMC algorithm parameters $e_{est\,\max}, T_1, T_2, \beta, \eta$ and ε, this paper suggests determining the optimal parameter vector $\mathbf{K}^*_{\text{ADRC-SMC1}} = [e^*_{est\,\max}\ T_1^*\ T_2^*\ \beta^*\ \eta^*\ \varepsilon^*]^T$, viewed as a vector variable in the optimization problem

$$\begin{aligned}\mathbf{K}^*_{\text{ADRC-SMC}} = & \underset{\mathbf{K}\in D_{\mathbf{K}_{\text{ADRC-SMC}}}}{\arg\min}\ J_{\mathbf{K}_{\text{ADRC-SMC}}}(\mathbf{K}_{\text{ADRC-SMC}}),\ J_{\mathbf{K}_{\text{ADRC-SMC}}}(\mathbf{K}_{\text{ADRC-SMC}}) = \\ & \int_{t_0}^{t_f} (e(t, \mathbf{K}_{\text{ADRC-SMC}}))^2 dt,\end{aligned} \tag{38}$$

where $[t_0, t_f]$ is the time horizon, $J_{\mathbf{K}_{\text{ADRC-SMC}}}$ is the cost function, $\mathbf{K}^*_{\text{ADRC-SMC}} = [e^*_{est\,\max}\ T_1^*\ T_2^*\ \beta^*\ \eta^* \varepsilon^*]^T$ is the solution to the optimization problem and the optimal gain matrix, $e(t)$ is the control error and $D_{\mathbf{K}_{\text{ADRC-SMC}}}$ is the feasible domain of $\mathbf{K}_{\text{ADRC-SMC1}}$. This feasible domain will be set as follows using the recommendations given in this section:

(i) The proper value of the design parameter $e_{est\,\max} > 0$ fulfills (23) and accounts for the dynamics important operating regimes of the control system.

(ii) The proper values of the design parameters $T_1 > 0,\ T_2 > 0$, and $\beta > 0$ and will impose the proper behavior of the control system on the sliding hyper-plane with the equation $\sigma(t) = 0$ using the expression of $\sigma(t)$ given in (35).

(iii) The proper values of the convergence factor $\eta > 0$ and the boundary layer thickness $\varepsilon > 0$ given in (28) respect the recommendations given in relation to (s51) given in [27] and (32), namely large values of $\eta > 0$ and small values of $\varepsilon > 0$ to guarantee the fulfillment of the sliding mode reaching and existence condition given in (20) (guaranteeing the control system stability) and to alleviate the effects of the chattering phenomenon.

The information included in the recommendations (i), (ii) and (iii) is formulated in terms of several inequality-type constraints embedded in the feasible domain of $\mathbf{K}_{\text{ADRC-SMC1}}$

$$\begin{aligned}D_{\mathbf{K}_{\text{ADRC-SMC}}} = \{ & [e_{est\,\max}\ T_1\ T_2\ \beta\ \eta\ \varepsilon] \in \Re^6 \mid e_{est\,\max} \in (0.0001, 0.1),\ T_1 > 0,\ T_2 > 0, \\ & \beta \in (0.0001, 10), \eta > 0, \varepsilon \in (0.0001, 10), 2\beta T_2 e_{est\,\max} < \eta \min(1, |\sigma(t)|/\varepsilon)\}.\end{aligned} \tag{39}$$

The dynamic regime involved in the closed-loop experiment needed to evaluate $J_{\text{K}_{\text{ADRC-SMC}}}$ in the optimization problem defined in (38) is the nominal operating regime of the control system. More details regarding the dynamic regime are given in Section 4.

The design strategy of the second-order continuous-time ADRC-SMC algorithm is summarized in the following steps:

Step ADRC-SMC-1. Specify the dynamic regime involved in the open-loop experiment and solve the optimization problem defined in (s5) given in [27] accounting for (s6) given in [27] to compute the optimal ELSO gain matrix $\mathbf{L}^* = [l_1^*\quad l_2^*\quad l_3^*\quad]^T$.

Step ADRC-SMC-2. Choose the value $b_0 \neq 0$ of the estimate of the coefficient of the control input.

Step ADRC-SMC-3. Set the value of the parameter $T_{DLP} > 0$ of the first-order derivative plus low-pass filter with the transfer function in (3) to implement the computation of $\tilde{\dot{r}}(t)$, $\tilde{\ddot{r}}(t)$, $\tilde{\dot{y}}(t)$ and $\tilde{\dot{e}}(t)$ in the control law illustrated in Fig. 2. This parameter value is set as a compromise to noise reduction and the delay induced by the filter.

Step ADRC-SMC-4. Specify the dynamic regime involved in solving the optimization problem in (38) and the evaluation of the cost function $J_{\mathbf{K}_{\text{ADRC-SMC}}}$, and solve the optimization defined in (38) accounting for (39) to compute the optimal parameter vector $\mathbf{K}^*_{\text{ADRC-SMC}} = [e^*_{est\,\max}\, T^*_1\, T^*_2\, \beta^*\, \eta^* \varepsilon^*]^T$.

The steps ADRC-SMC-1, ADRC-SMC-2 and ADRC-SMC-3 correspond to the ADRC part of the ADRC-SMC algorithm, therefore they are identical to the steps ADRC-1, ADRC-2 and ADRC-3, respectively. The last step, namely ADRC-SMC-4, corresponds to the SMC part of the ADRC-SMC algorithm.

3. The Experimental Validation

The validation of the theoretical results presented in Section 2 starts with applying, for the sake of comparison, steps ADRC-1 to ADRC-4 to design the ADRC. Next, steps ADRC-SMC-1 to ADRC-SMC-4 are applied to design the ADRC-SMC. All experiments are conducted on the tower crane system laboratory equipment by controlling the position of the cart, the arm, and the payload. In the following, to simplify the notations, the vector subscript attributed to cart position is 1, the vector subscript attributed to arm angular position is 2 and the vector subscript attributed to payload position is 3.

Step ADRC-1 and step ADRC-SMC-1 in the design of the ADRC and ADRC-SMC, respectively, requires setting the dynamic regime involved in the open-loop experiment. In cart position control, a chirp signal with 0.7 (PWM duty cycle) amplitude is applied; in arm angular position control, a chirp signal with 0.5 (PWM duty cycle) amplitude is applied; in payload position control, a chirp signal with 0.7 (PWM duty cycle) amplitude is applied. All signals for the three degrees of freedom of the tower crane system have 0.1 Hz as the initial frequency, 2 Hz as the frequency at the target time, and the target time is 45 s. The width of the time horizon involved in the open-loop experiments is of 35 s. The collected data is used in solving the optimization problem defined in (s5) given in [27] accounting for (s6) given in [27] to compute the optimal ELSO gain matrix $\mathbf{L}^* = [l^*_1 \quad l^*_2 \quad l^*_3 \quad]^T$.

SMA is applied to solve the optimization problem defined in (s5) given in [27]. For the sake of simplicity, all versions of SMA applied in this paper in both observer tuning and controller tuning are built around the implementation and the parameters taken from [9, 10, 28] and also given in [27].

In ELSO parameter tuning, the particular values of the parameters and the variables in the context of (s57) and (s58) given in [27] are $q = 3$, $\boldsymbol{\sigma} = \mathbf{L}$, $\boldsymbol{\sigma}^* = \mathbf{L}^*$ and $J = J_{\mathbf{L}}$. In this regard, the resulting optimal parameters of the ELSO gain matrices for the cart position, the arm angular position, and the payload position of the real-time tower crane system process are

$$\begin{aligned}\mathbf{L}_1^* &= [l_{11}^* \quad l_{12}^* \quad l_{13}^* \quad]^T = [157.70 \quad 8289.67 \quad 145250.17]^T,\\ \mathbf{L}_2^* &= [l_{21}^* \quad l_{22}^* \quad l_{23}^* \quad]^T = [151.10 \quad 7515.70 \quad 123018.21]^T,\\ \mathbf{L}_3^* &= [l_{31}^* \quad l_{32}^* \quad l_{33}^* \quad]^T = [157.10 \quad 8226.08 \quad 143565.52]^T.\end{aligned} \tag{40}$$

The corresponding values of the cost functions are $J_{\mathbf{L}_1}(\mathbf{L}_1) = 1.3929 \cdot 10^{-5}$, $J_{\mathbf{L}_2}(\mathbf{L}_2) = 3.7011 \cdot 10^{-4}$ and $J_{\mathbf{L}_3}(\mathbf{L}_3) = 1.2389 \cdot 10^{-5}$. These optimal ELSOs actually ensure the convergence because $\lim_{t\to\infty} |y(t) - \hat{y}(t)| = 0$, which is highlighted in Fig. 4 for the cart position, in Fig. 5 for the arm angular position, and in Fig. 6 for the payload position are given in [27]. Figs. 4 to 6 also highlight the ELSO convergence because $\lim_{t\to\infty} |\dot{y}(t) - \hat{z}_2(t)| = 0$.

Step ADRC-2 and step ADRC-SMC-2 are applied in terms of first choosing the value $b_0 \neq 0$ of the estimate of the coefficient of the control input. In this control application, the chosen value is $b_0 = 1.5$. The last common step in designing the ADRC and ADRC-SMC is represented by step ADRC-3 and step ADRC-SMC-3. Here, the filter time constant in (3) is set to $T_{DLP} = 0.001$ s.

The reference trajectories used for the experimental setup are given in [27] and zero initial conditions are assumed in all experiments. Therefore, the dynamic regimes in the optimization problems are fully specified.

In the case of the ADRC, step ADRC-4 is dedicated to the computation of the optimal parameter vector $\mathbf{K}_{\text{ADRC}}^*$ via an SMA algorithm, which solves the optimization problem defined in (5) accounting for (s7) given in [27]. The Multi Input-Multi Output (MIMO) control system structure is considered, i.e. the design of the three position controllers (cart position, arm angular position and payload position) is carried out simultaneously. The particular values of the parameters and the variables in the context of (s57) and (s58) given in [27] are $q = 6$, $\boldsymbol{\sigma} = \mathbf{K}_{\text{ADRC}}$, $\boldsymbol{\sigma}^* = \mathbf{K}_{\text{ADRC}}^*$ and $J = J_{\mathbf{K}_{\text{ADRC}}}$. In this regard, the obtained optimal parameters of the ADRC are

$$\mathbf{K}_{\text{ADRC}}^* = [K_{11}^* \;\; K_{12}^* \;\; K_{21}^* \;\; K_{22}^* \;\; K_{31}^* \;\; K_{32}^*]^T = [12.11 \;\; 9.10 \;\; 8.57 \;\; 5.80 \;\; 5.69 \;\; 8.63]^T. \tag{41}$$

In the case of the ADRC-SMC, step ADRC-SMC-4 is dedicated to the computation of the optimal parameter vector $\mathbf{K}_{\text{ADRC-SMC}}^*$ via an SMA algorithm, which solves the optimization problem defined in (38) accounting for (39). The MIMO control system is considered in this case as well. The particular values of the parameters and the variables in the context of (s57) and (s58) given in [27] are $q = 18$, $\boldsymbol{\sigma} = \mathbf{K}_{\text{ADRC-SMC}}$, $\boldsymbol{\sigma}^* = \mathbf{K}_{\text{ADRC-SMC}}^*$ and $J = J_{\mathbf{K}_{\text{ADRC}}}$. In this regard, the obtained optimal parameters of the ADRC-SMC are

$$\begin{aligned}\mathbf{K}_{\text{ADRC-SMC}}^* &= [e_{1est\max}^* \, T_{11}^* \, T_{12}^* \, \beta_1^* \eta_1^* \varepsilon_1^* \;\; e_{2est\max}^* \, T_{21}^* \, T_{22}^* \, \beta_2^* \eta_2^* \varepsilon_2^* \;\; e_{3est\max}^* \, T_{31}^* \, T_{32}^* \, \beta_3^* \eta_3^* \varepsilon_3^*]^T\\ &= [0.001 \; 5 \; 0.3 \; 0.4 \; 12 \; 0.0001 \; 0.001 \; 7 \; 0.15 \; 0.15 \; 12 \; 0.001 \; 0.05 \; 3 \; 0.4 \; 0.4 \; 15 \; 0.009]^T.\end{aligned} \tag{42}$$

The experimental results presented in Fig. 7 for the cart position, in Fig. 8 for the arm angular position, and in Fig. 9 for the payload position are illustrated after averaging ten sets of real-time experimental results are given in [27]. The purpose of giving averages of the experimental results is to reduce the effect of random disturbances that can appear at any time. The average and the variance values of the cost functions of the control systems with the ADRC and ADRC-SMC are presented in Table 1 is given in [27].

4. Conclusions

This paper proposed the second-order *Active Disturbance Rejection Control* (ADRC)-*Sliding Mode Control* (SMC) algorithm, resulted from mixing the second-order ADRC algorithm with an SMC algorithm. The main advantage of this mix is to improve the overall control-loop system performance and to guarantee its stability.

The parameters of the continuous-time ADRC-SMC algorithm were optimally tuned using metaheuristic *Slime Mould Algorithms* (SMAs), and the reaching and existence conditions of SMC are expressed as inequalities that guarantee control system stability and formulated as inequality-type constraints in the optimization problems solved by SMAs. Stability conditions were included in two theorems, and useful design recommendations were given. In addition, the parameters of the extended Luenberger state observers were optimally tuned using SMAs after the definition of the optimization problems.

The paper also suggested novel implemented control laws specific to ADRC and ADRC-SMC. These control laws are different to the theoretical ones by including the dynamics of the filters that produce the estimated derivatives of the reference inputs.

The performance of the control systems with ADRC-SMC algorithms was compared with that of the control systems with ADRC algorithms. Real-time experimental results in the position control of tower crane laboratory equipment were included to support the comparison.

Future research will be focused on the stability analysis of the control systems with the implemented control laws specific to ADRC and ADRC-SMC; the stability conditions to be derived in this regard will lead to different inequality-type constraints. In addition, the process models and the controllers will be implemented in discrete time, in different configurations assisted by appropriate analyses. Nevertheless, different optimization algorithms will be adapted and implemented, for instance, those successfully applied to PI and PID controllers tuning [29], nonlinear optimal control [30], manifold optimization in communication systems [31], fuzzy rule interpolation [32], motion primitives [33], fuzzy control [34, 35], human well-being and resilience [36], partitioning problems [37], haptic interfaces [38], medical image analysis [39], cyber-physical production systems [40], parameter adaptation of neural networks [41], disturbance observers in fuzzy systems [42], suspension systems [43], and networked control systems [44].

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