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Correlation-Aware Minimum Mean Square Error Channel Estimation for Uplink Massive Multiple-Input Multiple-Output Systems in Sixth-Generation Networks

Muhammad Ahsan SHAIKH[1], Sadiq Ur REHMAN[2,*], and Halar MUSTAFA[3]

[1]Department of Technology, Faculty of Engineering Sciences and Technology, Hamdard University, Karachi, Pakistan
[2]Department of Computer Science, Faculty of Engineering Sciences and Technology, Iqra University, Karachi, Pakistan
[3]Department of Electrical Engineering, Faculty of Engineering Sciences and Technology, Hamdard University, Karachi, Pakistan
Email: muhammad.ahsan@hamdard.edu.pk, sadiq.rehman@iqra.edu.pk*, halar.mustafa@hamdard.edu.pk
* Corresponding author

Abstract. Massive Multiple-Input Multiple-Output (mMIMO) is a key enabling technology for beyond fifth-generation (B5G) and sixth-generation (6G) wireless systems; however, its performance critically depends on accurate channel state information (CSI). Conventional minimum mean square error (MMSE) channel estimators are typically derived under the assumption of spatially uncorrelated fading, which rarely holds in practical deployments due to antenna coupling, limited spacing, and structured propagation environments. As a result, spatial correlation can significantly degrade estimation accuracy. This paper investigates uplink channel estimation for mMIMO systems under spatially correlated channels. First, an analytical characterization of the effect of correlation on the traditional MMSE estimator is presented. Then, two correlation-aware estimation techniques are developed. The first approach introduces a regularized MMSE formulation to improve robustness in correlated scenarios. The second approach proposes a pre-whitening-based MMSE estimator that exploits channel covariance information to transform the correlated channel into an equivalent white channel prior to estimation. Closed-form solutions for the optimal filters are derived, and their complexity is calculated. The results of the simulation verify that the pre-whitening based estimator outperforms the conventional and regularized MMSE estimators over a wide interval for the value of the signal to noise ratio (SNR) and the correlation coefficient.

Key-words: Channel estimation; massive MIMO; MMSE; 6G.

1. Introduction

Future wireless networks are expected to support data-intensive and latency-critical applications such as extended reality (XR), holographic communications, ultra-reliable low-latency communications (uRLLC), and massive machine-type communications (mMTC). As discussed in [1], sixth-generation (6G) systems are expected to deliver extremely high data rates and ultra-low latency to enable such applications. In contrast, [2] provides a comprehensive overview of fifth-generation (5G) systems and highlights their limitations in meeting these stringent requirements. Recent studies such as [3] analyze beamforming strategies in Multiple-Input Multiple-Output Non-Orthogonal Multiple Access (MIMO-NOMA) systems, while [4] reviews emerging beamforming techniques for beyond fifth-generation (B5G) systems. Additionally, [5] investigates outage performance under fading conditions. However, these works do not explicitly address channel estimation under spatial correlation.

Massive multiple-input multiple-output (mMIMO) technology has been widely recognized as a fundamental enabler for future wireless communication systems. By deploying a large number of antennas at the base station (BS), mMIMO enables simultaneous transmission to multiple users over the same time–frequency resources, thereby improving spectral efficiency and spatial multiplexing. For instance, [6] analyzes the impact of signal-to-interference-plus-noise ratio (SINR) on system performance, while [7] investigates channel estimation strategies in mMIMO systems. Despite these advancements, the performance gains of mMIMO systems strongly depend on the availability of accurate channel state information (CSI) at the BS.

CSI usually follows from minimum mean square error estimation through pilots under Gaussian distribution assumptions. The comparison between traditional and machine learning-based channel estimation is presented in [8]. Nevertheless, most minimum mean square error algorithms assume independent fading, which is unrealistic in practical wireless systems.

In reality, spatial correlation arises due to the limited distance between antennas, mutual coupling effect, and weak scattering effect. Spatial correlation affects the precision of channel estimation and creates interference, as described in [9]. In addition, most spatial correlation aware schemes, like [10], have to directly invert the covariance matrix, making them numerically unstable under high correlation or weak SNR.

Inspired by such drawbacks, the present research seeks to analyze the problem of estimating a spatially correlated uplink channel of the mMIMO system with robust MMSE methodology. To this end, a approach utilizing the pre-whitening operation that can convert the correlated channel model to an uncorrelated one prior to estimation is proposed.

In addition, several recent works address the issue of robust estimation. In particular, the benefits of using a nonlinear estimation technique, such as the Extended Kalman Filter or Takagi-Sugeno fuzzy observer, as discussed in [11], lie in its ability to accommodate nonlinearity. At the same time, the role of optimization in ensuring system robustness is addressed in [12]. The significance of adaptive and stable estimation strategies is demonstrated in [13] and [14].

The major contributions of this paper are summarized as follows:

- Investigation of the effect of spatial correlation on MMSE channel estimation in mMIMO.

- Proposing a pre-whitening MMSE estimator that compensates for the deterioration in performance due to spatial correlation.
- Proposing a regularized MMSE estimator under the condition of high correlation and low SNR.
- Comprehensive simulation-based performance evaluation under varying correlation levels, antenna sizes, and user loads.

The structure of the paper is outlined as follows. Section 2 provides the system model. Section 3 reviews conventional MMSE estimation. Section 4 introduces the proposed estimator. Section 5 presents simulation results, followed by conclusions in Section 6.

2. System Model

Consider the uplink of a mMIMO system, where a BS equipped with M antennas serves K single-antenna users. We assume a typical mMIMO regime, i.e., $M \gg K$ [3]. The BS receives superimposed pilot signals from all users simultaneously.

Let $\mathbf{y}_t \in \mathbb{C}^{M\times 1}$ denote the received signal vector at the BS at the t^{th} time instant, as illustrated in Fig. 1 of [15]. The received signal is expressed as

$$\mathbf{y}_t = \sum_{i=1}^{K} \mathbf{h}_i x_{i,t}^* + \mathbf{n}_t, \tag{1}$$

where $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ represents the channel vector between the i^{th} user and the BS, $x_{i,t} \in \mathbb{C}$ is a scalar pilot symbol, and $\mathbf{n}_t \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_M)$ is the additive white Gaussian noise (AWGN) vector with $\sigma_n^2 \in \mathbb{R}^+$. The channel coefficient between the k^{th} user and the m^{th} BS antenna is modeled as

$$h_{k,m} = \sqrt{g_k}\, \tilde{h}_{k,m}, \tag{2}$$

where $\tilde{h}_{k,m} \sim \mathcal{CN}(0,1)$ denotes small-scale fading and $g_k \in \mathbb{R}^+$ is a scalar large-scale fading coefficient, assumed constant across antennas for each user. By stacking N received pilot symbols, the received signal matrix is written as

$$\mathbf{Y} = \mathbf{H}\mathbf{X}^H + \mathbf{N}, \tag{3}$$

where $\mathbf{Y} \in \mathbb{C}^{M\times N}$, $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_K] \in \mathbb{C}^{M\times K}$, $\mathbf{X} = [\mathbf{x}_1\ \cdots\ \mathbf{x}_K] \in \mathbb{C}^{N\times K}$ with $\mathbf{x}_k \in \mathbb{C}^{N\times 1}$, and $\mathbf{N} \in \mathbb{C}^{M\times N}$ is the noise. Assuming orthogonal pilot sequences satisfying $\mathbf{X}^H\mathbf{X} = \mathbf{I}_K$ [3], post-multiplying (3) by $\mathbf{X}$ yields

$$\mathbf{Z} \triangleq \mathbf{Y}\mathbf{X} = \mathbf{H} + \tilde{\mathbf{N}}, \tag{4}$$

here $\tilde{\mathbf{N}} \in \mathbb{C}^{M\times K} = \mathbf{N}\mathbf{X}$, $\mathbf{Z} \in \mathbb{C}^{M\times K}$. The k^{th} column of $\mathbf{Z}$ is given by

$$\mathbf{z}_k = \mathbf{h}_k + \tilde{\mathbf{n}}_k, \quad k = 1, \ldots, K, \tag{5}$$

with $\mathbf{z}_k, \tilde{\mathbf{n}}_k \in \mathbb{C}^{M\times 1}$. The channel estimate for the k^{th} user is obtained by linearly filtering $\mathbf{z}_k$ as,

$$\hat{\mathbf{h}}_k = \mathbf{W}_k^H \mathbf{z}_k, \tag{6}$$

where $\mathbf{W}_k \in \mathbb{C}^{M\times M}$ is the estimation matrix. The performance of the estimator depends on statistical parameters of the channel and noise, which are obtained from long-term observations. The large-scale fading coefficient is estimated as

$$g_k = \frac{1}{T}\sum_{t=1}^{T} \|\mathbf{z}_k(t)\|^2. \tag{7}$$

The spatial correlation matrix is estimated using sample covariance as

$$\mathbf{R}_k = \frac{1}{T}\sum_{t=1}^{T} \mathbf{h}_k(t)\mathbf{h}_k^H(t), \tag{8}$$

where $\mathbf{R}_k \in \mathbb{C}^{M\times M}$, this matrix is assumed to vary slowly and is updated over large time intervals.

The noise power is estimated during pilot-free intervals or using minimum eigenvalue-based estimation techniques in which λ_i^{noise} denotes small eigenvalues of the received covariance matrix and can be represented by

$$\sigma_n^2 \approx \frac{1}{M}\sum_{i=1}^{M} \lambda_i^{\text{noise}}. \tag{9}$$

A regularization matrix is introduced for numerical stability given by $\mathbf{\Pi} = \lambda \mathbf{I}_M$, having $\mathbf{\Pi} \in \mathbb{C}^{M\times M}$ and $\lambda \in \mathbb{R}^+$ is a tuning parameter.

3. MMSE-Based Channel Estimation

This section reviews the conventional MMSE channel estimation framework for uplink mMIMO systems and analytically highlights its limitations under spatially correlated channels, which motivate the proposed correlation-aware approach.

We first consider the case of spatially uncorrelated channels. In this scenario, the channel vector of the k^{th} user is expressed as $\mathbf{h}_k = \sqrt{g_k}\,\tilde{\mathbf{h}}_k$,, where $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$, $g_k \in \mathbb{R}^+$ denotes the large-scale fading coefficient (scalar), and $\tilde{\mathbf{h}}_k \in \mathbb{C}^{M\times 1}$ with $\tilde{\mathbf{h}}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ represents the normalized small-scale fading vector. The corresponding channel covariance matrix is given by

$$\mathbb{E}\big[\mathbf{h}_k\mathbf{h}_k^H\big] = g_k \mathbf{I}_M, \tag{10}$$

where $\mathbf{I}_M \in \mathbb{C}^{M\times M}$ is the identity matrix. Based on the pilot observation $\mathbf{z}_k = \mathbf{h}_k + \tilde{\mathbf{n}}_k$, where $\mathbf{z}_k \in \mathbb{C}^{M\times 1}$ and $\tilde{\mathbf{n}}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I}_M)$ with $\sigma_n^2 \in \mathbb{R}^+$, the MMSE estimator aims to minimize the mean square error (MSE)

$$J_k = \mathbb{E}\left[\left\|\hat{\mathbf{h}}_k - \mathbf{h}_k\right\|^2\right]. \tag{11}$$

The channel estimation problem can be formally posed as an optimization problem in which the objective is to determine the optimal linear estimator that minimizes the MSE. Specifically, for the k^{th} user, the problem is defined as

$$\mathbf{W}_k^{\star} = \arg \min_{\mathbf{W}_k \in \mathbb{C}^{M \times M}} \mathbb{E}\left[\left\|\mathbf{W}_k^H \mathbf{z}_k - \mathbf{h}_k\right\|^2\right], \quad \text{s.t. } \mathbb{E}[\mathbf{h}_k \mathbf{h}_k^H] = g_k \mathbf{R}_k. \tag{12}$$

Here $\mathbf{W}_k$ is the optimization variable. The objective minimizes the estimation error under known second-order channel statistics, forming the basis of the MMSE estimation framework.

Using the linear estimator $\hat{\mathbf{h}}_k = \mathbf{W}_k^H \mathbf{z}_k$ and standard trace-based manipulation, the MSE can be expressed as

$$\begin{aligned} J_k = \mathrm{Tr}\Big[& g_k \mathbf{W}_k^H \mathbf{W}_k - g_k \mathbf{W}_k^H - g_k \mathbf{W}_k + g_k \mathbf{I}_M \\ & + \sigma_n^2 \mathbf{W}_k^H \mathbf{W}_k\Big], \end{aligned} \tag{13}$$

where all matrix dimensions are $M \times M$. Differentiating J_k with respect to $\mathbf{W}_k^H$ and setting the result to zero yields the optimal MMSE weight matrix

$$\mathbf{W}_k = \frac{g_k}{\sigma_n^2 + g_k} \mathbf{I}_M, \tag{14}$$

and the corresponding minimum achievable MSE

$$J_{k,\min} = g_k M - \frac{g_k^2 M}{\sigma_n^2 + g_k}. \tag{15}$$

In practical deployments, antenna elements at the base station experience spatial correlation. In this case, the channel covariance matrix becomes

$$\mathbb{E}\big[\mathbf{h}_k \mathbf{h}_k^H\big] = g_k \mathbf{R}_k, \tag{16}$$

where $\mathbf{R}_k$ denotes the spatial correlation matrix. Substituting (16) into the MSE expression yields

$$J_k = \mathrm{Tr}\big[(\mathbf{W}_k^H - \mathbf{I}_M)\, g_k \mathbf{R}_k\, (\mathbf{W}_k - \mathbf{I}_M) + \sigma_n^2 \mathbf{W}_k^H \mathbf{W}_k\big]\,. \tag{17}$$

The above expression indicates that the performance of the MMSE estimator depends on the channel correlation matrix. In particular, when a conventional MMSE estimator designed under the assumption of spatially uncorrelated channels is applied in correlated environments, performance degrades due to model mismatch and the possible ill-conditioning of $\mathbf{R}_k$. This effect is illustrated in Fig. 2 [15], where correlated channels exhibit significantly higher MSE compared to spatially white channels.

4. Proposed MMSE-Based Channel Estimation Technique

This section presents a detailed derivation of the proposed MMSE-based channel estimation framework for spatially correlated mMIMO channels. We begin with the conventional MMSE estimator for correlated channels, followed by a regularized variant, and finally introduce the proposed pre-whitening–based MMSE estimator, which forms the core contribution of this work.

4.1. Conventional MMSE estimation for correlated channels

For the correlated channel model, the MMSE weight matrix $\mathbf{W}_k$ is obtained by minimizing the cost function defined in (17). The optimal solution is found by taking the gradient of the cost function with respect to $\mathbf{W}_k^H$ and setting it to zero, i.e.,

$$\frac{\partial J_k}{\partial \mathbf{W}_k^H} = \mathbf{0}. \tag{18}$$

This yields

$$g_k \mathbf{R}_k \mathbf{W}_k - g_k \mathbf{R}_k + \sigma_n^2 \mathbf{I}_M \mathbf{W}_k = \mathbf{0}. \tag{19}$$

Rearranging (19), the conventional MMSE solution for correlated channels is obtained as

$$\mathbf{W}_k = \left(g_k \mathbf{R}_k + \sigma_n^2 \mathbf{I}_M\right)^{-1} g_k \mathbf{R}_k. \tag{20}$$

Substituting (20) into the objective function, the minimum achievable MSE is given by

$$\begin{aligned} J_{k,\min} = g_k^3 T_r \operatorname{Tr}\Big[\mathbf{R}_k \left(g_k \mathbf{I}_M + \sigma_n^2 \mathbf{R}_k^{-1}\right)^{-2}\Big] - 2 g_k^2 T_r \operatorname{Tr}\Big[\mathbf{R}_k \left(g_k \mathbf{I}_M + \sigma_n^2 \mathbf{R}_k^{-1}\right)^{-1}\Big] \\ + \sigma_n^2 g_k^2 T_r \operatorname{Tr}\Big[\left(g_k \mathbf{I}_M + \sigma_n^2 \mathbf{R}_k^{-1}\right)^{-2}\Big] + g_k \operatorname{Tr}[\mathbf{R}_k]\,. \end{aligned} \tag{21}$$

It is evident from (21) that when $\mathbf{R}_k = \mathbf{I}_M$, the above expression reduces to the minimum MSE corresponding to the spatially white channel case, as reported in (12). However, for strongly correlated channels, the performance of the conventional MMSE estimator degrades due to the ill-conditioning of $\mathbf{R}_k$.

4.2. Algorithm 1: Regularized MMSE estimation

To improve numerical stability under strong spatial correlation, a regularization term is incorporated into the conventional MMSE cost function, yielding

$$J_k = T_r \mathbf{W}_k^H \mathbf{\Pi} \mathbf{W}_k + \mathbb{E}\left[\left\|\hat{\mathbf{h}}_k - \mathbf{h}_k\right\|^2\right]. \tag{22}$$

$\mathbf{\Pi}$ entries typically lie in the interval $[0, 1]$. The matrix $\mathbf{\Pi}$ may be adaptively tuned based on long-term channel statistics. Following a procedure analogous to the conventional MMSE derivation, the cost function in (22) can be expressed as

$$J_k = T_r \Big[\mathbf{W}_k^H \mathbf{\Pi} \mathbf{W}_k + (\mathbf{W}_k^H - \mathbf{I}_M) g_k \mathbf{R}_k (\mathbf{W}_k - \mathbf{I}_M) + \sigma_n^2 \mathbf{W}_k^H \mathbf{W}_k\Big]. \tag{23}$$

Taking the gradient with respect to $\mathbf{W}_k^H$ and setting it to zero yields the regularized MMSE solution

$$\mathbf{W}_k = \left(\mathbf{\Pi} + g_k \mathbf{R}_k + \sigma_n^2 \mathbf{I}_M\right)^{-1} g_k \mathbf{R}_k. \tag{24}$$

Algorithm 1 is obtained by solving the regularized MMSE optimization problem in 22 using gradient-based minimization.

Algorithm 1 Regularized MMSE channel estimation

1: **for** $k = 1$ to K **do**
  2: Compute $\mathbf{W}_k = (\mathbf{\Pi} + g_k\mathbf{R}_k + \sigma_n^2\mathbf{I}_M)^{-1}g_k\mathbf{R}_k$
  3: Estimate $\hat{\mathbf{h}}_k = \mathbf{W}_k^H\mathbf{z}_k$
4: **end for**

4.3. Algorithm 2: Proposed pre-whitening-based MMSE estimation

While regularization enhances numerical stability, it does not explicitly eliminate the impact of spatial correlation. To address this limitation, a pre-whitening–based MMSE channel estimation technique is proposed. The main idea is to transform the correlated channel observation into an equivalent whitened domain where MMSE estimation can be performed more reliably.

An artificial noise component $\mathbf{N}_2$ is first added to the received signal $\mathbf{Y}$ such that the effective noise $\bar{\mathbf{N}} = \mathbf{N}_1 + \mathbf{N}_2$ has the correlation property

$$\mathbb{E}\big[\bar{\mathbf{n}}_k\bar{\mathbf{n}}_k^H\big] = \sigma_n^2\mathbf{R}_k. \tag{25}$$

Accordingly, the modified received signal can be written as

$$\bar{\mathbf{Y}} = \mathbf{H}\mathbf{X}^H + \bar{\mathbf{N}}. \tag{26}$$

To remove the spatial correlation, the received signal is pre-whitened by right-multiplying $\bar{\mathbf{Y}}$ with $\mathbf{R}_k^{-H/2}$, yielding

$$\mathbf{Y}_{\text{new}} = \mathbf{R}_k^{-H/2}\bar{\mathbf{Y}} = \mathbf{R}_k^{-H/2}\mathbf{H}\mathbf{X}^H + \mathbf{R}_k^{-H/2}\bar{\mathbf{N}}. \tag{27}$$

Multiplying $\mathbf{Y}_{\text{new}}$ by the pilot matrix $\mathbf{X}$ gives

$$\mathbf{Z} = \mathbf{Y}_{\text{new}}\mathbf{X} = \mathbf{R}_k^{-H/2}\mathbf{H} + \mathbf{R}_k^{-H/2}\bar{\mathbf{N}}\mathbf{X}. \tag{28}$$

Hence, the k^{th} column of $\mathbf{Z}$ can be expressed as

$$\mathbf{z}_k = \mathbf{R}_k^{-H/2}\mathbf{h}_k + \mathbf{R}_k^{-H/2}\tilde{\mathbf{n}}_k, \tag{29}$$

where $\tilde{\mathbf{n}}_k$ denotes the effective noise after pilot projection. The channel estimate for the k^{th} user is then obtained using a linear MMSE filter as

$$\hat{\mathbf{h}}_k = \mathbf{W}_k^H\mathbf{z}_k. \tag{30}$$

Using the second-order statistics $\mathbb{E}[\mathbf{h}_k\mathbf{h}_k^H] = g_k\mathbf{R}_k$ and $\mathbb{E}[\tilde{\mathbf{n}}_k\tilde{\mathbf{n}}_k^H] = \sigma_n^2\mathbf{R}_k$, the corresponding MSE cost function can be simplified to

$$J_k = T_r\Big[(\mathbf{W}_k^H - \mathbf{R}_k^{-H/2})g_k(\mathbf{W}_k - \mathbf{R}_k^{-1/2}) + \sigma_n^2\mathbf{W}_k^H\mathbf{W}_k\Big]. \tag{31}$$

The optimal weight matrix is obtained by differentiating J_k with respect to $\mathbf{W}_k^H$ and equating the result to zero, which leads to

$$(g_k + \sigma_n^2)\mathbf{W}_k = g_k\mathbf{R}_k^{H/2}. \tag{32}$$

Algorithm 2 Proposed pre-whitening-based MMSE estimation

1: **for** $k = 1$ to K **do**
  2: Compute $\mathbf{W}_k = \dfrac{g_k}{g_k + \sigma_n^2}\mathbf{R}_k^{H/2}$
  3: Estimate $\hat{\mathbf{h}}_k = \mathbf{W}_k^H \mathbf{z}_k$
4: **end for**

Solving for $\mathbf{W}_k$, the optimal pre-whitening–based MMSE solution is given by

$$\mathbf{W}_k = \frac{g_k}{g_k + \sigma_n^2}\mathbf{R}_k^{H/2}. \tag{33}$$

The proposed estimation algorithms rely on statistical properties such as the channel covariance matrix $\mathbf{R}_k$, the large-scale fading gain g_k, and the noise variance σ_n^2, all computed from long-term channel statistics. Algorithm 1 includes a regularization factor to improve numerical stability, while Algorithm 2 applies a pre-whitening factor based on $\mathbf{R}_k$. These parameters are robust since they depend on slowly varying channel characteristics rather than instantaneous change.

4.4. Computational complexity and comparison

The proposed algorithm is benchmarked against a recently reported channel estimation approach for uplink mMIMO systems from [10] that estimate of the channel vector corresponding to the k^{th} user is given by

$$\hat{\mathbf{h}}_k = \left(\boldsymbol{\phi}_k^H \otimes \mathbf{I}_M\right)(\mathbf{D} \otimes \mathbf{I}_M)^{-1}\,\mathrm{vec}(\mathbf{Y}), \tag{34}$$

where $\boldsymbol{\phi}_k$ denotes the orthogonal pilot sequence assigned to the k-th user, $\mathbf{Y}$ represents the received signal matrix, and $\otimes$ denotes the Kronecker product. The matrix $\mathbf{D}$ is defined as

$$\mathbf{D} = \sigma^2\mathbf{I}_K + \sum_{(i,j)} \boldsymbol{\phi}_{i,j}\boldsymbol{\phi}_{i,j}^H, \tag{35}$$

with σ^2 denoting the noise variance and $\mathbf{I}_K$ being the $K \times K$ identity matrix. Computational complexity of order polynomial of all used approaches is almost similar, dominated by matrix inverse operation and matrix-vector multiplications. In order to make sure there is fair comparison between all approaches, all of them are simulated using same conditions for the simulation, namely, same range of SNR, same channel realizations, same pilot designs and same system specifications.

4.5. Impact of log-normal shadowing

To incorporate large-scale fading effects, the channel gain g_k is modeled as

$$g_k = (\alpha_k\beta_k)^2, \tag{36}$$

where α_k represents distance-dependent path loss and β_k models log-normal shadowing. Specifically,

$$\alpha_k = \sqrt{\frac{L_k}{d_k^\tau}}, \tag{37}$$

with d_k denoting the distance between the BS and the kth user, τ the path-loss exponent, and L_k the reference power loss. The log-normal shadowing component is given by

$$\beta_k = 10^{\frac{\sigma_k v_k}{10}}, \tag{38}$$

where $v_k \sim \mathcal{N}(0, 1)$ and σ_k denotes the shadowing standard deviation in dB.

5. Results and Analysis

To clarify how the theoretical framework is applied in the validation section, the simulation setup directly implements the analytical models developed in the previous sections. In each Monte Carlo iteration, correlated channel realizations are generated according to the assumed statistical model, and the received pilot signals are constructed using the system model. The conventional MMSE estimator, the regularized MMSE estimator, and the proposed pre-whitening-based MMSE estimator are then applied exactly as derived in the optimization framework to estimate the channel vectors. The estimated channels are compared with the real channel realizations to compute the normalized mean square error (NMSE), and the final results are obtained by averaging over a large number of independent realizations. All approaches are tested under identical simulation settings, including channel conditions, pilots, and SNR, with comparable computational complexity to ensure a fair comparison. The NMSE is used as the performance metric and is defined for the k^{th} user as

$$\text{NMSE}_k \triangleq \frac{\mathbb{E}\left[\|\hat{\mathbf{h}}_k - \mathbf{h}_k\|^2\right]}{M g_k}. \tag{39}$$

5.1. Effect of channel correlation

To analyze the impact of spatial correlation, the channel correlation matrix is modeled as [16]

$$\mathbf{R} = \begin{bmatrix} 1 & \alpha_c & \alpha_c^2 & \cdots & \alpha_c^{M-1} \\ \alpha_c & 1 & \alpha_c & \cdots & \alpha_c^{M-2} \\ \alpha_c^2 & \alpha_c & 1 & \cdots & \alpha_c^{M-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_c^{M-1} & \alpha_c^{M-2} & \alpha_c^{M-3} & \cdots & 1 \end{bmatrix}, \tag{40}$$

where $\alpha_c \in [0, 1]$ denotes the spatial correlation coefficient. The NMSE performance for $\alpha_c = 0.5$ is present in Fig. 3 [15]. It can be observed that the proposed Algorithm 2 achieves noticeably lower NMSE compared to the existing algorithm, particularly in the low SNR region ranging from 0 dB to 12 dB. This improvement indicates that the pre-whitening approach is effective in mitigating moderate channel correlation.

It can be seen that Fig. 4 [15] shows the performance comparison for a highly correlated channel scenario with $\alpha_c = 0.95$. In this case, it can be said that the suggested Algorithm 2 performs better than the existing algorithm throughout the whole range of SNRs from 0 dB to 18 dB. Additionally, it should be noted that as the value of the correlation coefficient rises from $\alpha_c = 0.5$ to $\alpha_c = 0.95$, the improvement in the suggested algorithm's performance becomes more significant.

5.2. Effect of number of users

This subsection investigates how changing the number of users influences channel estimation accuracy. Fig. 5 of [15] shows the relationship between the SNR value and NMSE with a fixed number of users, $K = 8$. Naturally, an increase in SNR results in the decline of NMSE for both techniques, which implies the enhancement of estimation accuracy owing to higher signal quality. Nevertheless, the suggested technique shows superior performance compared to the state-of-the-art approach throughout the whole range of SNRs. This advantage becomes more significant at higher values of SNR because Algorithm-2 makes better use of the available signal quality and provides good performance even with the increased number of users.

In Fig. 6 in [15], the performance of NMSE is shown for the increased number of users, which is 16. Albeit the increased number of users raises the NMSE for both algorithms compared to the 8 users case, due to an increase in the multi-user interference, the proposed Algorithm-2 outperforms the existing algorithm at all levels of SNR.

Further increasing the number of users to 32, as depicted by Fig. 7 in [15], leads to further degradation of performance for both algorithms because of amplification of multiuser interference (MUI). It is, however, clear that Algorithm 2 performs better than the existing algorithm, especially at low SNR values, highlighting its robustness under high user density.

5.3. Effects of path loss

The effect of path loss on channel estimation performance is analyzed by varying the path loss factor L_k between 0.1 and 0.9, with the corresponding NMSE results depicted in Fig. 8 in [15].

Clearly, it can be observed from the graphs that the NMSE value increases with the reduction of path loss value, indicating poor estimation. But interestingly, it is observed that as the SNR is increased, the NMSE curves become closer to each other, especially when SNR value is greater than 15 dB. Most significantly, the proposed Algorithm 2 performs better than the current algorithm for all values of path loss. To be more specific, for SNR value of 9 dB and $L_k = 0.1$, the NMSE value of current algorithm is around 0.72, while that of proposed algorithm is just around 0.6.

5.4. Effect of number of transmission antennas

The impact of the number of antennas used for transmission on the NMSE performance of the proposed Algorithm 2 is shown in Fig. 9 in [15]. The number of antennas used for transmission is taken as two values, which are 32 and 256 with a constant number of users as 16.

It is seen from both figures that the NMSE falls with an increase in SNR, which means that the performance of channel estimation becomes better. However, the NMSE for the case of 256 antennas is slightly larger than the case when 32 antennas are employed. This means that although there is improvement in channel estimation due to the increase in the number of antennas, there is also some degradation due to the complexity in estimation.

5.5. Comparison of all algorithms

Fig. 10 in [15] shows a comparative study of the proposed and the existing channel estimation techniques, where the approach described in [10] is included, based on the relationship between

NMSE and SNR for $K = 8$ users. As expected, NMSE reduces as SNR increases for all cases, which means better estimation performance. Nevertheless, the proposed approach provide better results than the existing techniques.

In particular, the proposed Algorithm 2 achieves the lowest NMSE among all compared schemes, with the most significant performance gains observed in the low SNR region. These results clearly demonstrate that the enhancements introduced in the proposed approach lead to more accurate and robust channel estimation, especially under challenging operating conditions.

6. Conclusions

Accurate channel estimation is a critical requirement for mMIMO systems envisioned for future 6G networks. In this paper, two MMSE-based channel estimation techniques were proposed for correlated mMIMO channels: a regularized MMSE estimator and a pre-whitening-based MMSE estimator.

Simulation results demonstrate that both proposed techniques significantly outperform existing correlated-channel estimation approaches in terms of NMSE. Among them, the pre-whitening-based MMSE algorithm achieves the best performance, particularly under strong spatial correlation, low-SNR conditions, and high user density.

Furthermore, the impact of spatial correlation, number of users, path loss with log-normal shadowing, and antenna array size was investigated. The results confirm that channel estimation performance degrades with increasing user count due to enhanced multiuser interference, while the proposed algorithm maintains robust performance across a wide range of system configurations.

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