

An Adapted Bounded Fuzzy Stepwise Weight Assessment Ratio Analysis (MF SWARA) Method

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Abstract. Determining criteria weights represents a significant step in multi-criteria decision-making (MCDM) field, as weights of the criteria directly influence the ranking of alternatives and the robustness of final decisions. Among subjective methods for determining the weights of the criteria, the Stepwise Weight Assessment Ratio Analysis (SWARA) method has been adopted for solving problems in various areas due to its simplicity, applicability and strong interpretability. To address uncertainty and vagueness inherent in human judgments, several fuzzy extensions of the SWARA method have been proposed. Existing SWARA extensions often generate intermediate parameters that exceed the unit interval, which complicates interpretability and reduces numerical transparency. Therefore, this paper proposes a modified fuzzy extension of the SWARA method that ensures all parameters obtained during

the calculation procedure of the fuzzy SWARA method to remain within the interval $[0, 1]$. To validate the proposed approach, several numerical illustrations are presented, including both individual and group decision making scenarios, followed by computational simulation implemented in Python programming language. The conducted numerical examples and simulation results confirm that the proposed extension achieves results identical to those obtained by the improved multiplicative weighting (IMW) SWARA method, with the note that the computational procedure of the proposed extension ensures that all variable values are in the interval $[0, 1]$. As such, the proposed MF SWARA method represents a bounded, theoretically consistent and a valuable advancement in the ongoing fuzzy MCDM weighting methods.

Key-words: Criteria weights; fuzzy sets; MCDM; SWARA; triangular fuzzy numbers.

1. Introduction

Determining the weights of the criteria is crucial in decision-making (DM), having in mind that even a slight change in weights can have a huge impact on how alternatives are ranked [1]. Determining the weights of the criteria constitutes an indispensable step in DM process, as it fundamentally shapes the evaluation structure [2] and directly influences final decision outcomes [3–5]. For this reason, a large number of Multiple-Criteria Decision-Making (MCDM) methods for determining the weights of criteria have been developed and successfully applied in practice, such as: Entropy method [6], Analytic Hierarchy Process (AHP) method [7], Best-Worst (BWM) method [8], Method Based on the Removal Effects of Criteria (MEREC) [8, 9], and so forth.

The methods for determining the weight of the criteria belong to one of the mentioned categories [10]: a) objective methods where criteria weights are determined based on input data from the decision matrix, that is, based on the performances of the alternatives concerning the criteria, obtained from one or more decision-makers, and b) subjective methods, which enable decision makers to express their attitudes regarding the significance of criteria much more directly. Step-wise Weight Assessment Ratio Analysis (SWARA) method is proposed by Kersulienė et al. [11]. This method can be used for the complete solving of an MCDM problem, but it is often used in combination with other methods, where the SWARA method is used only for determining criteria weights. As some more significant examples of the combined application of the SWARA method and some other MCDM methods, the following can be mentioned: [12–15].

In order to apply the SWARA method for solving significantly more complex decision-making problems, this method was adapted for the use of fuzzy numbers [16, 17]. In contrast to classical sets, where an element belongs or does not belong to a set, fuzzy set theory, introduced by Zadeh [18], enables partial belonging to a set, where membership in a set is expressed by applying the membership function $\mu_{\tilde{A}}(x)$, where $\mu_{\tilde{A}}(x) \in [0, 1]$. In fuzzy decision-making, numerical values are expected to preserve a clear semantic interpretation. Thus, ensuring bounded fuzzy parameters enhances interpretability, facilitates communication of results to decision-makers, and improves confidence in the obtained weights.

The proposed fuzzy extensions of the SWARA method are based on the application of triangular fuzzy numbers (TFNs), which enable the parameters that define the boundaries of the triangle, i.e. the left boundary, mode and right boundary, to have values greater than 1. That is why the values of the parameters obtained by applying the calculation procedure of these fuzzy extensions of the SWARA method are, in some cases, greater than 1. Therefore, this paper proposes a modification of the fuzzy SWARA method, in which the values of the parameters during

the calculation belong to the interval $[0, 1]$. The proposed modification gives the same results as the existing fuzzy extensions of the SWARA method, with the fact that it is not proposed to replace the existing fuzzy extensions of the SWARA method, but to serve as a basis for proposing Intuitionistic, Pythagorean, Spherical or Neutrosophic extensions of the SWARA method. By introducing boundedness, an interpretability and numerical stability is enhanced, and provides a stable and consistent basis for the development of advanced fuzzy extensions. Therefore, the main contribution can be summarized as follows: a) a bounded extension of the fuzzy SWARA method is developed – which ensures that all computational elements remain within the interval $[0, 1]$; b) by introducing MF SWARA, weighting structure is preserved, ensuring full methodological compatibility; c) introduction of a flexible linguistic scale for MF SWARA which is adapted to bounded framework; d) different defuzzification procedures and their influence is examined.

The primary aim of the research is to provide a new modification of fuzzy SWARA (MF SWARA) method in which all intermediate and final parameters remain strictly within the interval $[0, 1]$, while preserving the whitening outcomes of existing fuzzy SWARA approaches. To present the proposed MF SWARA, the remaining part of this article is organized as follows: In Section 2, the fuzzy extension of the SWARA method is presented, while in Section 3 MF SWARA is proposed. In Section 4, the criteria weights are determined using the MF SWARA method presenting its potential. Finally, conclusions are given.

2. Preliminaries

2.1. The SWARA method

The procedure for determining the relative importance of an alternative using the SWARA method can be presented as follows [19]:

Step 1. Determine the evaluation criteria and arrange them in descending order based on their expected significance.

Step 2. Set the relative significance s_j of each criterion, except the first criterion, as follows:

$$s_j = \begin{cases} 0 & C_j = C_{j-1} \\ > 0 & C_j > C_{j-1} \end{cases}, \quad (1)$$

where C_j and C_{j-1} denote the significance of criterion j and criterion $j - 1$, respectively; and $j \neq 1$.

Step 3. Calculate the value of coefficient k_j as follows:

$$k_j = \begin{cases} 1 & j = 1 \\ 1 + s_j & j > 1 \end{cases}. \quad (2)$$

Step 4. Calculate the value of coefficient q_j as follows:

$$q_j = \begin{cases} 1 & j = 1 \\ \frac{q_{j-1}}{k_j} & j > 1 \end{cases}. \quad (3)$$

Step 5. Determine the weights of the evaluation criteria as follows:

$$w_j = \frac{q_j}{\sum_{k=1}^n q_k}, \quad (4)$$

where w_j denotes the weight of criterion j , and k denotes the number of the criteria. Based on (1), a higher value of s_j indicates a lower significance of criterion j regarding criterion $j - 1$. In general, the value s_j indicates how much less significant the criterion C_j is in relation to the previous criterion C_{j-1} . For example, when s_{j-1} a value of 0.1 has the significance of criteria C_{j-1} will be 10% lower than the significance of criterion C_j , as illustrated in Table 1.

Table 1. The meaning of s_j in the SWARA method

Criterion	s_j	k_j	q_j	w_j	%
C_1		1	1	0.284	
C_2	0.00	1.00	1.00	0.284	0.00%
C_3	0.10	1.10	0.91	0.259	-10.00%
C_4	0.50	1.50	0.61	0.172	-50.00%
			3.52	1.00	

2.2. Triangular fuzzy numbers

The fuzzy set (FS) theory was proposed by Zadeh [18], as an extension of a classical set theory. In contrast to the classical set theory, FS theory allows for partial membership to a set.

Definition 1. [20]. A fuzzy subset $\tilde{A}$ of a non-empty set X is defined as follows:

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x) \rangle \mid x \in X \}, \quad (5)$$

where $\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$ is the membership function of FS $\tilde{A}$, $\mu_{\tilde{A}}(x) \in [0, 1]$ is the degree of membership of the element x to the $\tilde{A}$. Membership functions can have different shapes such as bell-shaped, Gaussian, trapezoidal, and triangular.

Definition 2. [21]. Triangular fuzzy number (TFN) $\tilde{A}$ can be represented as a triplet $\tilde{A} = (l, m, u)$ where membership function is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - l)/(m - l) & l \leq x < m \\ 1 & x = m \\ (u - x)/(u - m) & m < x \leq u \\ 0 & \text{otherwise} \end{cases}, \quad (6)$$

where l , m , and u denote left endpoint, mode, and right endpoint, respectively, of TFN $\tilde{A}$, and $l < m < u$.

Definition 3. [22]. Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be a two positive TFNs, and k is a positive crisp number. The arithmetic operations of these TFNs, necessary for the development of the fuzzy extension of the SWARA method, are as follows:

$$\tilde{A} \oplus \tilde{B} = (a_l + b_l, a_m + b_m, a_u + b_u), \quad (7)$$

$$\tilde{A} \odot \tilde{B} = (a_l/b_u, a_m/b_m, a_u/b_l), \quad (8)$$

$$\tilde{A} \cdot k = (a_l \cdot k, a_m \cdot k, a_u \cdot k), \quad (9)$$

$$\tilde{A}/k = \left(a_l \cdot \frac{1}{k}, a_m \cdot \frac{1}{k}, a_u \cdot \frac{1}{k} \right), \quad (10)$$

$$\tilde{A} + k = (a_l + k, a_m + k, a_u + k). \quad (11)$$

2.3. The fuzzy SWARA and IMW SWARA methods

The process of defining criteria weights using the fuzzy SWARA method can be represented by the following steps [16, 17]:

Step 1. Determine the evaluation criteria and arrange them in descending order based on their expected significance.

Step 2. Set the relative significance $\tilde{s}_j$ of each criterion using fuzzy linguistic variables. Mavi et al. [16] proposed a fuzzy linguistic scale with five points, which is shown in Table 2.

Table 2. Fuzzy linguistic scale proposed by Mavi et al. [16]

Linguistic variable	TFN scale
Equally important	(1, 1, 1)
Moderately less important	(2/3, 1, 3/2)
Less important	(2/5, 1/2, 2/3)
Very less important	(2/7, 1/3, 2/5)
Much less important	(2/9, 1/4, 2/7)

The fuzzy linguistic variable ‘Equally important’ in Table 2 has a fuzzy value (1, 1, 1), which is equivalent to a crisp value 1. It is obvious that this is an inadequate fuzzy scale because in the SWARA method, “Equally important”, that is “Same importance” corresponds to the value 0, not 1. Therefore, Vrtagic et al. [17] proposed the application of a new fuzzy linguistic scale, which is shown in Table 3.

Table 3. Fuzzy linguistic scale proposed by Vrtagic et al. [17]

Linguistic variable	TFN scale
Equally significant	(0, 0, 0)
Weakly less significant	(2/9, 1/4, 2/7)
Moderately less significant	(1/4, 2/7, 1/3)
Less significant	(2/7, 1/3, 2/5)
Really less significant	(1/3, 2/5, 1/2)
Much less significant	(2/5, 1/2, 2/3)
Dominantly less significant	(1/2, 2/3, 1)
Absolutely less significant	(1, 1, 1)

Step 3. Calculate the value of fuzzy coefficient $\tilde{k}_j$ as follows:

$$\tilde{k}_j = \begin{cases} \tilde{1}, & j = 1, \\ \tilde{1} + \tilde{s}_j, & j > 1. \end{cases} \quad (12)$$

Step 4. Calculate the value of fuzzy coefficient $\tilde{q}_j$ as follows:

$$\tilde{q}_j = \begin{cases} \tilde{1}, & j = 1, \\ \frac{\tilde{q}_{j-1}}{\tilde{k}_j}, & j > 1. \end{cases} \quad (13)$$

Step 5. Determine fuzzy weights $\tilde{w}_j$ of the evaluation criteria as follows:

$$\tilde{w}_j = \frac{\tilde{q}_j}{\sum_{k=1}^n \tilde{q}_k}. \quad (14)$$

Step 6. Determine crisp weights of the evaluation criteria.

In Mavi et al. [16] and Vrtagic et al. [17], different procedures were suggested to convert fuzzy criteria weights into crisp values. Mavi et al. [16] use a very simple defuzzification procedure proposed by Opricovic and Tzeng [23]. In this approach, the left endpoint, mode, and right endpoint of TFN have the same significance:

$$df(\tilde{w}) = \frac{1}{3}(l + m + u). \quad (15)$$

In contrast, Vrtagic et al. [17] propose an approach in which the mode of TFN is more important than left and right endpoints:

$$df(\tilde{w}) = \frac{1}{6}(l + 4m + u). \quad (16)$$

3. The Modified Fuzzy SWARA (MF SWARA) Method

In the following, a modification of the fuzzy SWARA method is proposed, which enables all the values of the coefficients $\tilde{k}_j$ and $\tilde{q}_j$, as well as the value of the $\sum_{k=1}^n \tilde{q}_k$ to, remain within the interval $[0, 1]$. The process of determining the weights of the criteria using the modified fuzzy SWARA method can be presented using the following steps:

Step 1. Determine the evaluation criteria and arrange them in descending order based on their expected significance.

Step 2. Set the relative significance $\tilde{s}_j$ of each criterion using fuzzy linguistic variables. MF SWARA can be used with the linguistic variables shown in Table 3, but a new fuzzy linguistic scale, presented in Table 4, is also proposed for use with MF SWARA.

Table 4. Fuzzy linguistic scale proposed for using with MF SWARA

Linguistic variable	Abbreviation	TFN scale	Crisp value
Equally significant	ES	(0, 0, 0)	0.00
Slightly less significant	SL	(0, 0, 0.25)	0.08
Moderate less significant	ML	(0, 0.25, 0.5)	0.25
Less significant	LS	(0.25, 0.5, 0.75)	0.50
Very less significant	VL	(0.5, 0.75, 1)	0.75
Much less significant	ML	(0.75, 1, 1)	0.92

The MF SWARA method can be applied using other linguistic scales. Crisp values shown in the column named Crisp values were calculated using (13). Based on the values from this column, it can be seen that the proposed scale is not linear. Linguistic scales significantly facilitate the application of fuzzy numbers, but the choice of one of the linguistic variables offered, and transformation of its meaning into a certain TFN, can to some extent limit the expression

of the respondents' attitudes regarding the significance of the criteria. In the case of applying this modification of the SWARA method, it is recommended that they can modify the predefined values of TFN scales following their preferences when they consider it necessary.

Step 3. Calculate the value of fuzzy coefficient $\tilde{k}_j$ as follows:

$$\tilde{k}_j = \begin{cases} (0.5, 0.5, 0.5), & j = 1, \\ \frac{(\tilde{1} + \tilde{s}_j)}{2}, & j > 1. \end{cases} \quad (17)$$

Step 4. Calculate the value of fuzzy coefficient $\tilde{q}_j$ as follows:

$$\tilde{q}_j = \begin{cases} (0.5, 0.5, 0.5), & j = 1, \\ \frac{\tilde{q}_{j-1}}{2\tilde{k}_j}, & j > 1. \end{cases} \quad (18)$$

Step 5. Calculate the recalculated value $\tilde{q}'_j$ of fuzzy coefficient $\tilde{q}_j$ as follows:

$$\tilde{q}'_j = \frac{\tilde{q}_j}{n}. \quad (19)$$

Step 6. Determine fuzzy weights $\tilde{w}_j$ of the evaluation criteria as follows:

$$\tilde{w}_j = \frac{\tilde{q}'_j}{\sum_{k=1}^n \tilde{q}'_k}. \quad (20)$$

Step 7. Determine crisp weights of the evaluation criteria.

Step 5 in the MF SWARA is introduced with the aim of ensuring that $\sum_{k=1}^n \tilde{q}_k$ is less than 1.

3.1. Determining crisp weights of the criteria in the MF SWARA

Defuzzification of the fuzzy criteria weights in the MF SWARA method can be performed similarly as in the fuzzy SWARA and IMW SWARA methods by using (15) or (16).

However, in the case of using linguistic variables that are transformed into symmetrical TFNs, in cases of decision-making based on the attitudes of one DM, the results obtained by applying fuzzy MCDM methods can be as the results obtained by using classic MCDM methods. The difference, between the criteria weights obtained applying the fuzzy SWARA method applied with linguistic variables represented with symmetrical fuzzy numbers and the SWARA method applied with crisp numbers is illustrated in the supplementary material [24].

Significantly greater benefits from applying fuzzy extensions can be achieved by using defuzzification procedures which enables giving different significances to the boundaries and modes of TFNs, such as using the following equations:

$$df(\tilde{w}) = \frac{\alpha l + m + (1 - \alpha)u}{2}, \quad (21)$$

$$df(\tilde{w}) = \frac{\alpha l + \lambda m + (1 - \alpha)u}{2}. \quad (22)$$

In Eqs. (21) and (22) coefficient $\alpha \in [0, 1]$ and coefficient $\lambda \geq 0$.

In the case of applying Eqs. (15), (16), (21) and (22), the sum of the calculated weights can be greater than 1, which is why it is necessary to normalize criteria weights as follows:

$$w_j = \frac{w_j}{\sum_{k=1}^n w_k}. \quad (23)$$

3.2. Determining weights of the criteria using MF SWARA in group decision-making

There are two basic scenarios for determining the criteria weights in group decision-making, i.e.: a) the first, less frequently used, approach in which criteria weights are calculated individually for each respondent and based on these individual criteria weights, group criteria weights are determined usually as the average of the individual criteria weights. In this approach, it is possible to assign different importance to respondents; and b) the second approach is based on forming group attitudes, usually group decision-making matrix, based on the individual attitudes of respondents, and perform further calculation using group decision-making matrix. It is recommended that the group MF SWARA method include both scenarios, whereby the determination of the criteria weights is actually based on the second approach, that is, the use of a group decision-making matrix. In other words, the group MF SFARA method has two basic scenarios.

The first basic scenario. In the first basic scenario, the criteria weights were determined individually for each respondent, using the MF SWARA method, whereby the respondents were suggested to check the values of the boundaries and modes of the TFNs after the transformation of the linguistic variables into TFNs, and correct them according their preferences, with the fulfillment of the conditions $0 < l \leq m \leq u \leq 1$. In cases where respondents are not familiar with the use of MCDM methods and/or fuzzy numbers, it is recommended that they be provided with assistance from persons who are skilled in the application of fuzzy extensions of MCDM methods. It is also suggested the usage of forms created in a spreadsheet software for collecting the opinions of respondents, so that the calculation can be carried out interactively, and the results, i.e. criteria weights, can be presented numerically and graphically after each modification. By using such approach, the respondents could more easily notice whether the obtained values of the criteria weights realistically reflect their attitude, and otherwise, they could correct the linguistic variables with which they express their attitudes regarding the significance of the criteria or the values of the boundaries and mode corresponding TFNs.

The second basic scenario. The second scenario of using MF SWARA method in the group decision-making environment is based on the application of the calculation procedure of the MF SWARA method using a group decision-making matrix, which is formed based on the opinions of the respondents obtained in the previous step. After determining the fuzzy criteria weights, crisp criteria weights are determined using one of the previously presented defuzzification procedures.

4. Numerical Illustrations

The purpose of the conducted numerical illustrations is to demonstrate applicability, effectiveness, consistency and robustness of the proposed MF SWARA method. Considered examples are designed to highlight the preservation of the weighting structure and the boundedness of the

intermediate parameters, as well as influence of different decision-making scenarios in individual and group contexts. Therefore, in order to verify, as well as emphasize the specificity of the proposed approach, three numerical illustrations and one computational simulation are presented in this section: a) the first numerical illustration presents the use of the proposed approach for calculating criteria weights for the evaluation of rural tourism trips based on the opinions of one respondent; b) the second numerical illustration presents the application of the MF SWARA method in a group decision-making environment, also for determining criteria weights; c) the third numerical illustration shows a comparison of the results obtained using the IMF SWARA and MF SWARA methods (provided as supplementary material [24]), and the computational simulation also presents a comparison of the results obtained using the IMF SWARA and MF SWARA methods (provided as supplementary material [24]).

4.1. The first numerical illustration

As mentioned previously, the first numerical illustration presents the use of the proposed approach for calculating criteria weights for evaluating rural tourism trips based on the opinions of one respondent. This example is partially adopted from Stanujkic et al. [25]. In this numerical illustration MF SWARA method is used for determining criteria weights of the following criteria: destination's appeal (C_1); Supporting capacities (C_2); Accommodation and amenity (C_3); Transportation and availability (C_4), and Price (C_5).

The opinions of respondent concerning the significance of the criteria are shown in Table 5, while the calculation details are presented Table 6.

Table 5. Attitudes of respondents

Criterion	Linguistic variable	Abbreviation	$\tilde{s}_j$		
			l	m	u
C_1					
C_2	Less significant	LS	0.25	0.50	0.75
C_3	Equally significant	ES	0.00	0.00	0.00
C_4	Slightly less significant	SL	0.00	0.00	0.25
C_5	Equally significant	ES	0.00	0.00	0.00

Table 6. Calculation details and fuzzy criteria weights obtained using MF SWARA method

Criterion	$\tilde{k}_j$			$\tilde{q}_j$			$\tilde{q}'_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.50	0.50	0.50	0.50	0.50	0.50	0.10	0.10	0.10	1.19	1.36	1.64
C_2	0.63	0.75	0.88	0.29	0.33	0.40	0.06	0.07	0.08	0.68	0.91	1.31
C_3	0.50	0.50	0.50	0.29	0.33	0.40	0.06	0.07	0.08	0.68	0.91	1.31
C_4	0.50	0.50	0.63	0.23	0.33	0.40	0.05	0.07	0.08	0.54	0.91	1.31
C_5	0.50	0.50	0.50	0.23	0.33	0.40	0.05	0.07	0.08	0.54	0.91	1.31
							0.31	0.37	0.42			

The values in columns $\tilde{k}_j$, $\tilde{q}_j$, $\tilde{q}'_j$ and $\tilde{w}_j$ were calculated using Eqs. (17) to (20). Crisp values of criteria weights obtained using Eqs. (15) and (23) are shown in Table 7.

Table 7. Fuzzy and crisp values of criteria weights

Criterion	$\tilde{w}_j$			w_j
	l	m	u	
C_1	1.19	1.36	1.64	0.270
C_2	0.68	0.91	1.31	0.187
C_3	0.68	0.91	1.31	0.187
C_4	0.54	0.91	1.31	0.178
C_5	0.54	0.91	1.31	0.178

A similar process of transforming fuzzy to crisp criterion weights could be done by applying Eqs. (16), (21) or (22). The obtained results demonstrate that the proposed MF SWARA method produces a consistent distribution of criteria weights while ensuring that all computational elements remain within the predefined interval. The weights of the criteria reflect initial preferences of decision-maker, confirming that transformation does not distort the relative importance of the criteria structure.

4.2. The second numerical illustration

The second numerical illustration demonstrates the application of the MF SWARA method in a group decision-making environment. The attitudes of the first of the three respondents were previously shown in Table 5, while the attitudes of the remaining two respondents are shown in Tables 8 and 9.

Table 8. Attitudes of the second of the three respondents

Criterion	Linguistic variable	Abbreviation	$\tilde{s}_j$		
			l	m	u
C_1					
C_2	Slightly less significant	SL	0.00	0.00	0.30
C_3	Moderate less significant	ML	0.00	0.25	0.50
C_4	Slightly less significant	SL	0.00	0.00	0.35
C_5	Equally significant	ES	0.00	0.00	0.00

In the MF SWARA method, linguistic variables are used to help respondents to more easily transform their preferences regarding the importance of the criteria into a corresponding fuzzy number, where it is emphasized that respondents should check the result of the transformation of linguistic variables into fuzzy numbers and correct them following their preferences, with the fulfillment of the conditions $l \leq m \leq u \leq 1$. The second respondent used this possibility and, in Table 8, corrected the upper limit of the linguistic variable “Slightly less significant” to 0.35, instead of 0.30, for criterion C_4 .

The group fuzzy decision matrix formed on the attitudes of three respondents is presented in Table 10, while the details of the calculation of the fuzzy criteria weights are presented in Table 11.

Crisp weights obtained by the defuzzification procedures expressed by Eqs. (15), (16), (21) and (22), and also by applying Eq. (23), are shown in Table 12 and in Fig. 1.

Table 9. Attitudes of the third of the three respondents

Criterion	Linguistic variable	Abbreviation	$\tilde{s}_j$		
			l	m	u
C_1					
C_2	Equally significant	ES	0.00	0.00	0.00
C_3	Equally significant	ES	0.00	0.00	0.00
C_4	Slightly less significant	SL	0.00	0.00	0.25
C_5	Equally significant	ES	0.00	0.00	0.00

Table 10. Group attitudes of respondents

Criterion	$\tilde{s}_j$		
	l	m	u
C_1			
C_2	0.08	0.17	0.35
C_3	0.00	0.08	0.17
C_4	0.00	0.00	0.28
C_5	0.00	0.00	0.00

Table 11. Calculation details and fuzzy criteria weights obtained using group MF SWARA method

Criterion	$\tilde{k}_j$			$\tilde{q}_j$			$\tilde{q}'_j$			$\tilde{w}_j$		
	l	m	u	l	m	u	l	m	u	l	m	u
C_1	0.50	0.50	0.50	0.50	0.50	0.50	0.10	0.10	0.10	0.213	0.236	0.297
C_2	0.54	0.58	0.68	0.37	0.43	0.46	0.07	0.09	0.09	0.158	0.203	0.274
C_3	0.50	0.54	0.58	0.32	0.40	0.46	0.06	0.08	0.09	0.135	0.187	0.274
C_4	0.50	0.50	0.64	0.25	0.40	0.46	0.05	0.08	0.09	0.105	0.187	0.274
C_5	0.50	0.50	0.50	0.25	0.40	0.46	0.05	0.08	0.09	0.105	0.187	0.274
							0.34	0.42	0.47			

Table 12. Calculation details and fuzzy criteria weights obtained using group MF SWARA method

Criterion	$\tilde{w}_j$			w_j				$\bar{w}_j$
	l	m	u	(15)	(16)	(21)	(22)	
C_1	0.213	0.236	0.297	0.240	0.238	0.239	0.239	0.236
C_2	0.158	0.203	0.274	0.204	0.203	0.204	0.204	0.203
C_3	0.135	0.187	0.274	0.192	0.189	0.191	0.191	0.194
C_4	0.105	0.187	0.274	0.182	0.185	0.183	0.183	0.183
C_5	0.105	0.187	0.274	0.182	0.185	0.183	0.183	0.183

From Table 12, it can be noticed that by applying different defuzzification procedures, different criteria weights are obtained, as well as that these differences in weights are not so big. However, there are two valid reasons for this: a) the attitudes of the respondents do not show great differences in the significance of the criteria, and b) Eqs. (21) and (22) were used with the

following coefficients $\alpha = 0.5$ and $\lambda = 1$.

Column $\bar{w}_j$ of the previous table also shows the criteria weights, which represent the average value of the criteria weights obtained based on the attitudes of each respondent, calculated in the first step of the group MF SWARA method.

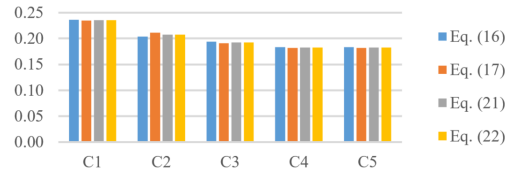


Fig. 1. Criteria weights obtained using different defuzzification procedures.

Significantly greater benefits from the use of defuzzification procedures represented by Eqs. (21) and (22) can be conceded by varying the values of coefficients α and λ . One example of applying Eq. (21) and different values of the coefficient α is presented in Table 13 and in Fig. 2.

Table 13. Criteria weights obtained using Eq. (21) and different values of coefficient α

Criterion	$\tilde{w}_j$			w_j		
	l	m	u	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
C_1	0.213	0.236	0.297	0.223	0.239	0.262
C_2	0.158	0.203	0.274	0.199	0.204	0.210
C_3	0.135	0.187	0.274	0.193	0.191	0.188
C_4	0.105	0.187	0.274	0.193	0.183	0.170
C_5	0.105	0.187	0.274	0.193	0.183	0.170

Further analysis of varying defuzzification parameters reveals that the MF SWARA method allows controlled sensitivity regarding weights of the criteria. By adjusting coefficients, decision-makers can highlight different aspects of the fuzzy numbers (central tendency of spread) without compromising the overall consistency. Such flexibility enhances adaptability of the method in various decision-making environments.

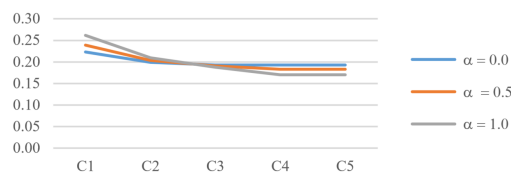


Fig. 2. Criteria weights obtained using Eq. (21) and different values of coefficient α .

5. Conclusions

This paper proposes a modification of the fuzzy SWARA method, IM SWARA, in which the values of all variables used in the calculation procedure applied for determining criteria weights

belong to the interval $[0, 1]$. The proposed modification gives the same results as the existing fuzzy extensions of the SWARA method, especially IMW SWARA method, which was proven by three numerical calculations and one computational simulation, written in the Python programming language. The key contribution of the paper lies in providing a bounded and consistent fuzzy SWARA framework that enhances interpretability, improves numerical stability and facilitates integration into large-scale or simulation-based decision support systems. Therefore, proposed method provides an adequate mythological foundation for further developments that requires strict parameter boundedness. It should be emphasized here that the newly proposed extension, that is MF SWARA method, is not proposed to replace the existing fuzzy extensions of the SWARA method, or more precisely the IMW SWARA method, but rather to serve as a basis for proposing intuitionistic, Pythagorean, spherical or neutrosophic extensions of the SWARA method. For this reason, attempts to develop and apply intuitionistic and neutrosophic extensions of the SWARA method can be mentioned as directions for the future development of the proposed modification.

References

- [1] S. KORUCUK and A. AYTEKİN, *Prioritization of criteria for production planning software selection using picture fuzzy PIPRECIA method: A case of the Ordu province*, Journal of Process Management and New Technologies **13**(3–4), 2025, pp. 97–106.
- [2] M. K. SRIVASTAVA, S. GAUR and A. OHRI, *Analysing the Effectiveness of MCDM and Integrated Weighting Approaches in Groundwater Quality Index Development*, Water Conservation Science and Engineering **9**, 2024, paper 35.
- [3] M. A. MOKTADIR, S. K. PAUL, C. BAI and E. D. SANTIBANEZ GONZALEZ, *The current and future states of MCDM methods in sustainable supply chain risk assessment*, Environment, Development and Sustainability **27**(3), 2025, pp. 7435–7480.
- [4] Z. GUO, J. LIU, X. LIU, Z. MENG, M. PU, H. WU, X. YAN, G. YANG, X. ZHANG, C. CHEN and F. CHEN, *An integrated MCDM model with enhanced decision support in transport safety using machine learning optimization*, Knowledge-Based Systems **301**, 2024, paper 112286.
- [5] Z. HASANKHANI, M. ZANDIEH and E. B. TIRKOLAEI, *Identification of optimal waste-to-energy strategies for sustainable development in Iran: SWOT analysis, hybrid MCDM methods, and game theory*, Renewable Energy **237**, 2024, paper 121564.
- [6] C. E. SHANNON, *A mathematical theory of communication*, The Bell System Technical Journal **27**(3), 1948, pp. 379–423.
- [7] L. T. SAATY, *The Analytic Hierarchy Process*, McGraw Hill Company, New York, NY, USA, 1980.
- [8] J. REZAEI, *Best-worst multi-criteria decision-making method*, Omega **53**, 2015, pp. 49–57.
- [9] M. KESHAVARZ-GHORABAEI, M. AMIRI, E. K. ZAVADSKAS, Z. TURSKIS and J. ANTUCHEVICIENE, *Determination of objective weights using a new method based on the removal effects of criteria (MERECE)*, Symmetry **13**(4), 2021, paper 525.
- [10] M. ŞAHİN, *Location selection by multi-criteria decision-making methods based on objective and subjective weightings*, Knowledge and Information Systems **63**(8), 2021, pp. 1991–2021.
- [11] V. KERŠULIENE, E. K. ZAVADSKAS and Z. TURSKIS, *Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA)*, Journal of Business Economics and Management **11**(2), 2010, 243–258.

- [12] M. ALIMARDANI, S. HASHEMKHANI ZOLFANI, M. H. AGHDAIE and J. TAMOŠAITIENĖ, *A novel hybrid SWARA and VIKOR methodology for supplier selection in an agile environment*, Technological and Economic Development of Economy **19**(3), 2013, pp. 533–548.
- [13] M. R. GHORSHI NEZHAD, S. H. ZOLFANI, F. MOZTARZADEH, E. K. ZAVADSKAS and M. BAHRAMI, *Planning the priority of high tech industries based on SWARA-WASPAS methodology: The case of the nanotechnology industry in Iran*, Economic Research-Ekonomska Istraživanja **28**(1), 2015, pp. 1111–1137.
- [14] J. HEIDARY DAHOOIE, E. BEHESHTI JAZAN ABADI, A. S. VANAKI and H. R. FIROOZ FAR, *Competency-based IT personnel selection using a hybrid SWARA and ARAS-G methodology*, Human Factors and Ergonomics in Manufacturing & Service Industries **28**(1), 2018, pp. 5–16.
- [15] H. PRAJAPATI, R. KANT and R. SHANKAR, *Prioritizing the solutions of reverse logistics implementation to mitigate its barriers: A hybrid modified SWARA and WASPAS approach*, Journal of Cleaner Production **240**, 2019, paper 118219.
- [16] R. K. MAVI, M. GOH and N. ZARBAKHSNIA, *Sustainable third-party reverse logistic provider selection with fuzzy SWARA and fuzzy MOORA in plastic industry*, The International Journal of Advanced Manufacturing Technology **91**, 2017, pp. 2401–2418.
- [17] S. VRTAGIĆ, E. SOFTIĆ, M. SUBOTIĆ, Ž. STEVIĆ, M. DORDEVIC and M. PONJAVIC, *Ranking road sections based on MCDM model: New improved fuzzy SWARA (IMF SWARA)*, Axioms **10**(2), 2021, paper 92.
- [18] L. A. ZADEH, *Fuzzy sets*, Information and Control **8**(3), 1965, pp. 338–353.
- [19] D. STANUJKIC, D. KARABASEVIC and E. K. ZAVADSKAS, *A framework for the selection of a packaging design based on the SWARA method*, Engineering Economics **26**(2), 2015, pp. 181–187.
- [20] P. A. EJEGWA, *Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition*, Complex & Intelligent Systems **5**(2), 2019, pp. 165–175.
- [21] C. B. CHENG, *Group opinion aggregation based on a grading process: A method for constructing triangular fuzzy numbers*, Computers & Mathematics with Applications **48**(10–11), 2004, pp. 1619–1632.
- [22] S. J. CHEN and C. L. HWANG, *Fuzzy Multiple Attribute Decision Making Methods: Methods and Applications*, Springer, Berlin, Heidelberg, 1992.
- [23] S. OPRICOVIC and G. H. TZENG, *Defuzzification within a multicriteria decision model*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems **11**(05), 2003, pp. 635–652.
- [24] D. STANUJKIC, D. KARABASEVIC, G. POPOVIC, A. ULUTAS, E. K. ZAVADSKAS and I. MEIDUTE-KAVALIUSKIENE, *An Adapted Bounded Fuzzy Stepwise Weight Assessment Ratio Analysis (MF SWARA) method, supplementary material ROMJIST*. Accessed: May 20, 2026 [Online]. Available: <https://dkarabasevic.com/supp-files/Supplementary-Material-ROMJIST.pdf>.
- [25] D. STANUJKIC, D. KARABASEVIC, G. POPOVIC, F. SMARANDACHE, P. S. STANIMIROVIĆ, M. SARAČEVIĆ and V. N. A. KATSIKIS, *A single valued neutrosophic extension of the simple WISP method*, Informatica **33**(3), 2022, pp. 635–651.